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Final Report

December 1978

# A METHOD FOR THE ALLOCATION OF EXPLORATORY DEVELOPMENT RESOURCES IN LOGISTICS

By: HENRY A. OLENDER

Prepared for:

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER BETHESDA, MARYLAND 20084

and

OFFICE OF NAVAL RESEARCH ARLINGTON, VIRGINIA 22217

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two of the measures of effectiveness at a time. The procedures then determine the implied preference of the decision maker between any pair of alternative programs through the consideration of a set of intermediate hypothetical programs described only in terms of their expected benefits. Several measures of effectiveness for the supply system were developed and modeled, and program alternatives were formulated to provide the basis for a numerical example. The values of the measures of effectiveness achievable by each program were estimated, and the procedures of the method were applied for illustrative purposes.

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#### PREFACE

This report documents the analysis and findings of a research project conducted for the David W. Taylor Naval Ship Research and Development Center (DTNSRDC), Bethesda, Maryland. The sponsor and technical monitor was M. J. Zubkoff, Code 187, of DTNSRDC. The Work was performed under contract N00014-77-C-0449, administered by the Office of Naval Research.

The research was performed in the Systems Evaluation Department (SED) of the Systems Research and Analysis Division (SRAD) of SRI International.

J. Naar is Director of SED; D. D. Elliott is Executive Director of SRAD.

H. A. Olender was the principle investigator. R. H. Monahan was project leader and assisted in the conduct of the research. H. B. Wilder, Jr., also provided technical assistance in the performance of the research.

#### I INTRODUCTION

#### A. Background

The David W. Taylor Naval Ship Research and Development Center (DTNSRDC) is the lead laboratory for naval logistics. In addition, the Technical Strategist for Logistics and Facilities is located there. It is the task of the Technical Strategist to develop and maintain an overall technical strategy to focus the thrust of all exploratory development (ED) in the field of naval logistics (certain specific logistic functions are assigned to other technical strategists). This approach to planning ED is innovative, especially for the area of naval logistics.

Naval logistics is heterogeneous, comprising a wide variety of very different and quite technical functions. These functions require different expertise, employ different technologies, and are evaluated by different measures of effectiveness. As a result, at the supporting establishment level (where most research and development is conducted), naval logistics has been largely planned, managed, and conducted in separate functional areas by separate agencies—e.g., Naval Supply Systems Command or Naval Sea Systems Command. For the most part, ED has been conducted according to the needs felt within each functional area with only broadbrush coordination among functional areas.

However, the Technical Strategist for Logistics is required to view naval logistics as a whole. He is to identify the regions of needed improvement, the pertinent emerging technologies to meet these needs, and the potential payoff in ED of technologies to meet the needs. Then he must recommend, from the alternative combinations of separate functional area ED programs, the integrated program that will result in the greatest benefit to overall naval logistics system effectiveness for the budget available for ED of logistics.

Much work remains to be done before the process of developing and maintaining a technical strategy for logistics is perfected. A pressing near-term requirement is a methodology for allocating ED resources among and within the key areas. A longer-term requirement is the development of a method to model the overall naval logistics system in order to measure the impact of changes in elements of the logistics system on fleet readiness or total system costs.

These two requirements are related. Proper allocation of the ED funds requires the knowledge of measures of effectiveness (MOEs) for the logistics system, and these MOEs are derived from the different steps required to model the overall naval logistics system.

Key tasks associated with resource allocation method are developing meaningful and useful MOEs and establishing explicit or implicit relationships (where they exist) among the various MOEs to better understand their impact on overall effectiveness; and developing a method of ED resource allocation for trading off the expected achievable levels of the MOEs that characterize each program.

#### B. The Problem

#### 1. Measures of Effectiveness

In each functional area of logistics, different MOEs have been defined to measure different aspects of performance--e.g., in the supply system, one MOE for operational performance is requisition fill rate, and for financial performance, one MOE is the ratio of sales to value of inventory. These are valid MOEs from the viewpoint of a supply officer at a supply depot. Different but related MOEs will be of concern to the user, such as an operational commander. He will be primarily concerned with the response time for the system to supply him with a certain type of part or quantity of material. This response time will be a function of, among other functional MOEs, the requisition fill rate mentioned above. Thus, no simple MOE is now, or may ever be, available to measure all important aspects of effectiveness for an entire functional area, and in some cases the MOEs used may be mutually conflicting.

Among different functional areas--e.g., the supply system and the maintenance system--the relationships between MOEs is even more ill-defined. Finally, in the overall system the interplay among different functional MOEs and their cumulative effect on the evaluation of the overall system effectiveness are only poorly defined.

#### 2. Resource Allocation for Exploratory Development

Currently the methods of arriving at the ED resource allocation decisions within the relatively short deadlines imposed by budget schedules depend mainly on judgment, experience, and intuition. Without a formal method for allocating ED resources among the heterogeneous key areas of logistics, the decisions are difficult to make and the rationale followed in the selection may be hard to reconstruct.

The difficulty lies in the fact that each key-area technology program is characterized by a set of expected achievable levels of different but important MOEs. Each of these characteristics or attributes measures a different type of effectiveness, and they cannot now be objectively and quantitatively traded off to determine the preferred program or the order of preference of other programs. A subjective methodology is required, which is based on judgmental inputs by decision-makers.

#### C. Research Objectives

The research reported herein was undertaken to provide an initial attack on these key resource allocation problems. Due to the complexity of the naval logistics system, the research focused on one key technical area--the supply system. However, the results should provide a basis for the development of models for each of the technical areas (and in the longer term for the overall naval logistics system) and for the establishment of resource allocation decision procedures.

The primary objectives of this research were:

 Identification and development of measures of effectiveness (MOEs) models for one key logistics technical area.

- (2) Development of a general resource allocation (RA) method for multi-attribute (i.e., disparate MOEs) outcome problems.
- (3) Application of the RA method developed in Item 2 to a sample problem in the technical area considered in Item 1. This sample problem will be based on realistic budget levels and ED program alternatives.

#### II SUMMARY AND CONCLUSIONS

#### A. Summary

A method for decision making in the allocation of funds for exploratory development (ED) programs was developed to specifically address the question of how to compare alternatives whose expected outcomes are multifaceted. The multifaceted aspect of the problem results from the fact that there is no single measure of effectiveness that can be used to judge the benefits of the many diverse technical functions comprising the logistic area. A decision maker (DM) is then faced with the problem of determining how different MOEs should be traded off to arrive at the most beneficial combination of expected outcomes.

The method relies heavily on the subjective but informed judgment of a DM. It assumes that the DM has a subjective model relating the needs of the Navy to fulfill its mission, the various logistics MOEs that relate to the Navy's capability to carry out this mission, and the relative effects of improvements in these MOEs on this capability. The method allows the DM to progressively build up and communicate his preferences concerning specific ED programs and their expected outcomes expressed as achievable levels of important MOEs. He does this through a sequence of MOE tradeoff assessments between two alternatives that differ only in the values of two MOEs. These tradeoff assessments result in the construction of a sequence of hypothetical alternatives that link two real alternatives, and allow the inference of a preference (or ranking) between these two alternatives. Systematically, applying this approach sequentially to all available alternatives results in a relative ranking among them.

As tradeoff assessment information is expressed and built up, a point can be reached where remaining alternatives may be analytically ranked without further tradeoff assessments. This last procedure is considered optional, depending on the confidence attained by the DM in

the adequacy of his expressed tradeoff assessment information as a basis for a valid analytical model.

The primary requirement of the method is the identification of a comprehensive set of MOEs that quantify the expected outcomes of the various constant-budget ED programs. All important MOEs relating the various logistics areas and affected by the contemplated ED programs and the mission of the Navy must be included. However, to the extent possible, the number of MOEs should be minimized to include only those that measure distinct and separate types of effectiveness. Required inputs to the procedure then consist of postulated alternative ED programs and the expected outcomes of the programs in terms of the MOE levels.

The RA method procedure employs a tableau where two real alternative programs are represented by their expected MOE values. These MOE values are entered in an order that facilitates use of the tableau format and application of the procedures. Tradeoff assessments by the DM are then used to construct a set of hypothetical alternatives in the tableau that link the two real alternatives in a manner that eventually allows the inference of a preference between the two real alternatives. Each hypothetical alternative differs from any preceding and successive alternative (whether real or hypothetical) in only two out of a total of n MOEs. This allows the DM to focus on his tradeoff between only two MOEs at a time.

The key concept of the method is that the set of hypothetical alternatives serve as surrogate alternatives for one of the real alternatives in the sense that the DM is indifferent between obtaining the sets of outcome MOEs for the real alternative and for any one of the hypothetical alternatives.

The RA method consists of the following steps:

- (1) Select alternative pair
- (2) Reorder MOEs
- (3) Construct tableau
- (4) Perform tradeoff assessment
- (5) Test for dominance
- (6) Complete the tradeoff assessments (optional)

- (7) Perform linear optimization for next alternative selection
- (8) Test for termination (optional).

Figure II-1 presents a flow chart of these steps. In the first step, a-priori subjective judgment is used to select a pair of alternatives for comparison. In Step 2, the MOEs are then reordered in the tableau to facilitate the remaining procedures. This will allow, in certain cases, the determination of the preference between the pair of alternatives without the need to perform all pairwise MOE tradeoff assessments. The tableau is then constructed (Step 3) and pairwise MOE tradeoff assessments (Step 4) are performed to construct the sequence of hypothetical alternatives. In Step 5, after some minimum number of tradeoff assessments have been made (a number than can be determined from the MOE reordering process), a test can be made to see if one of the real alternatives completely dominates (in terms of preference) the last hypothetical alternative. If this occurs, it can be inferred that this dominating alternative must be preferred to the other real alternative. This follows because each hypothetical alternative is constructed to be equal in preference to each other, and to the first real alternative.

If such a dominance does not occur, the performance of additional pairwise tradeoff assessments is required in accordance with Step 4, but after each new hypothetical alternative is constructed a test for dominance is conducted as per Step 5. Eventually dominance is achieved or the maximum required number of MOE tradeoff assessments have been completed. In the latter case, the second real alternative can be preferentially ranked relative to the last hypothetical alternative. Since the latter is a surrogate for the first real alternative, this preference equates to the preference between the two real alternatives.

Once preference is established between a pair of alternatives, and unranked alternatives still remain, the DM has an option on how to proceed. One approach is to return to Step 1, select a new pair of alternatives consisting of the current most preferred alternative and an arbitrary alternative from the remaining unranked alternatives, and iterate through the procedures again. If there are no more unranked alternatives,

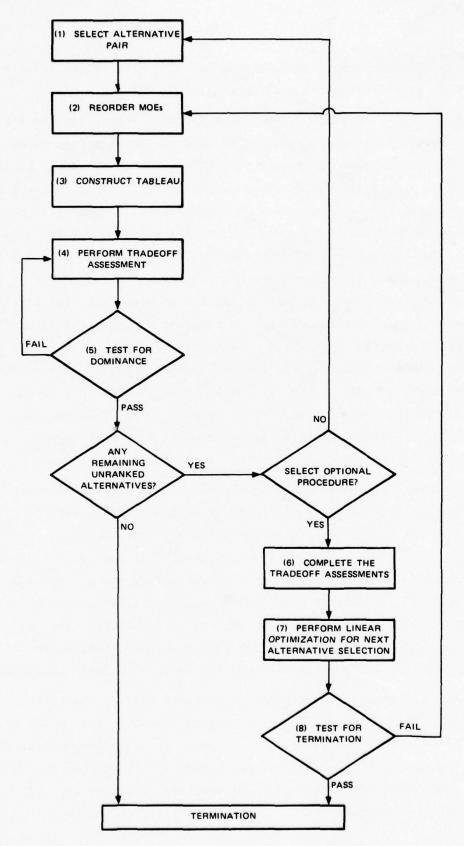


FIGURE II-1 RA METHOD FLOWCHART

then the procedure is terminated and the most preferred alternative has been identified.

The other approach is to proceed to Step 6, where the remaining tradeoff assessments, if required, are completed. This provides sufficient information for the construction of a linear model that locally approximates the DM's tradeoff structure. This model can then be used with an optimization procedure to determine a potentially most preferred alternative from those remaining unranked alternatives (Step 7).

Proceeding to Step 8, the DM has two alternatives to assess; the alternative derived in Step 7, and the current most preferred alternative. The linear model of Step 7 will also indicate a potential preference or rank between these two alternatives. If the DM has confidence in the validity of the linear model relative to these two alternatives, he may accept the results of the linear model, which will also provide the ranking of all remaining alternatives. Step 8 consists of the assessment of the validity of the linear model. If the validity is in question, the procedure continues by returning to Step 2 with the two alternatives identified above. Otherwise, the procedure is terminated, and the most preferred alternative has been identified.

The procedure is primarily based on the ability of the DM to perform Step 4, the tradeoff assessment between a pair of MOE values for two different alternatives. However, the only difference between the two alternatives is the values of the pair of MOEs being considered. For the tradeoff assessment, the DM is asked how much one MOE would have to be increased (or decreased) to compensate for a decrease (or increase) of a given amount in the other MOE. This given amount of decrease (or increase) is selected to equate the value of this MOE to its value for the second real alternative.

In addition to the method development, a demonstration problem was constructed. This problem considers the Logistic Supply System, one of the technical areas addressed in the Technical Strategy for Logistics and Facilities. An examination of this technical area resulted in the identification of three basic categories of MOEs: expected response time, costs, and manpower. The expected response time was taken as the time

from the requisition to receipt of a supply item or material. Costs and manpower are those increases or savings incurred in the implementation of the results of the ED programs. These basic MOE categories do not necessarily represent a comprehensive list of important MOE categories for the supply system. However, these were assumed to be adequate for the purpose of an initial effort on MOE selection and modeling, and a demonstration of the application of the method. In addition, it was also decided, for demonstration purposes, to focus on response time and costs only. Specific MOEs developed consisted of response times for several different supply scenarios, and two types of costs (or savings)—capital investment, and operations and maintenance, Navy (O&MN) costs. Each response time for a given scenario constitutes a separate MOE since no objective and quantitative models for relating these different response times were found. In addition, the two different types of cost were considered separately at the request of the sponsor.

The following seven MOEs were identified and used to demonstrate the RA method:

- (1)  $R_{A1}$  = Expected response time in Scenario A1
- (2)  $R_{\Delta 2}$  = Expected response time in Scenario A2
- (3)  $R_B$  = Expected response time in Scenario B
- (4)  $R_C$  = Expected response time in Scenario C
- (5)  $R_D$  = Expected response time in Scenario D
- (6) C<sub>T</sub> = Annualized capital investment costs
- (7)  $C_{OMN}$  = Annual operating and maintenance, Navy costs.

Scenario Al applies to the case of a small-parts requisition by a repair shop from a local supply point (LSP) or a nonlocal supply point (NLSP), if the part is not available at the LSP. Scenario A2 is similar in that it applies again to the case of a small-parts requisition, but onboard a ship at sea. In this case, the LSP is the ship's own supply stocks, and the NLSP is another ship's supply stocks. The second ship is assumed to be in the same task force as the ship requiring the repair part. These two scenarios are quite similar as their designation implies,

but they were kept separate because of the difference in urgency between the two cases. Scenario B is a heavy-equipment requisition from an NLSP by an aircraft maintenance shop or a shippard. Scenario C is a highvolume, normal supply requisition from an NLSP. Finally, Scenario D covers the case of underway replenishment at sea.

A responsiveness model was constructed to relate the outcomes of various proposed ED projects to the expected response time. This model was constructed to apply to all five different scenarios. Similarly, cost models were established to relate the outcomes of various ED projects to the two types of costs.

To construct alternative programs, three budget levels were considered. These were called incremental budget cases, and corresponded to 25%, 50%, and 75% of a maximum add-on and new-starts budget. At each of these incremental budget levels, several alternate combinations of ED projects were identified as feasible alternative programs. For each of these programs it was possible to identify projects, estimate outcome parameters, and employ these outcome parameters in the responsiveness and cost models to determine MOE values. The set of MOE values for each alternative forms the input to the RA method.

Finally, numerical estimates were made for the ED project outcomes. These estimates were not based on any significant analysis of the projects, but simply represent an example of what might occur. The RA method procedures were then applied to the example using the author in the role of the DM. The process was carried through only for the 25% budget to the point where all procedures were illustrated. A complete solution would require additional iterations, and the consideration of the two remaining incremental budget cases.

#### B. Conclusions

The RA method appears to be a viable means of trading off MOEs and ranking ED program alternatives, if the two following tasks are accomplished to provide the necessary inputs:

- (1) Selection and modeling of important MOEs and their relations to ED project outcomes.
- (2) Modeling and analysis of ED projects expected outcomes to obtain reasonably high confidence in estimates of their outcomes.

The utility of the RA method will depend strongly on the DM. If he finds that he is able to make the tradeoff assessments required in a consistent and confident manner, the method will provide him with:

- (1) A means of simplifying, organizing, and focusing his task of judging the merits of ED programs characterized by multiple and disparate MOEs.
- (2) A preference ranking among various ED program alternatives.
- (3) A means of tracing his rationale for ranking the alternatives, checking his judgmental consistence, and correcting inconsistencies as they are found.
- (4) A means of communicating his tradeoff and preference structure to his staff. This information can be employed to adjust or construct and evaluate new technology programs.

The attack on the problems of MOE selection and modeling has only been initiated in the effort reported herein. Whether the proposed RA method is implemented or not, this is an area that must be better defined and understood for improving resource allocation in all logistics technical areas. For example, in the supply area, the scenarios and responsiveness model can undoubtedly be improved, the manpower MOE should be addressed, and other MOEs such as supply throughput should be included.

Another significant problem area is the resources and techniques available for analysis of ED project expected outcomes. This problem interfaces with the MOE selection and modeling problem, since only after appropriate MOE models have been established can the important ED outcome parameters be identified and analysis techniques be developed to quantify them.

#### III A RESOURCE ALLOCATION METHODOLOGY

Resource allocation decisions must, in the final analysis, be based on an overall view of the effects of improvements in each of the various logistics functional areas on the ability of the Navy to carry out its assigned mission. There is no one single MOE that can measure that ability. The best we can do at any given level of decision making is to identify a number of meaningful and important MOEs that relate to fleet effectiveness, readiness, and costs.

Once these MOEs have been identified, ED programs can be developed that will improve each of the MOEs. Conversely, quantitative estimations of the improvement in each MOE expected as a result of successful completion of a given program can be used to compare alternative programs. If there were only one MOE, or if through an appropriate model one composite MOE could be developed to measure the relative effects of many MOEs, alternative ED programs could be ranked in terms of their expected output of that single MOE. Assuming further that each alternative program required the same budget, the resource allocation problem would be solved by simply choosing the alternative program with the highest rank.

When two MOEs comprise the irreducible set of important MOEs, the ranking task becomes more complex. Considering any two equal budget alternatives, we can expect one of several cases to occur. In the first case, one alternative may dominate the other in the expected level of both MOEs. Clearly, this alternative will be preferred or ranked above the other alternative. The second case occurs when one alternative dominates another in one MOE, but is equal in the other. For ranking purposes this is the same as Case 1, and we would rank the first alternative above the other.

A third case occurs when both alternatives are equal in both MOEs. Clearly one alternative is as good as the other. The choice can then be made based on other second order considerations. The final difficult case is where one alternative dominates another in one MOE but is dominated in the other MOE. It is no longer clear which alternative should be ranked higher, or whether they should be equally ranked. Their ranking will now depend on the relative importance of one MOE versus the other. A tradeoff assessment is required to determine how many units of one MOE are equivalent to one unit of the other MOE.

Assuming for the moment that this assessment is made for the two alternatives in question and one is ranked above the other, the consideration of a third alternative may or may not be simplified. If the relative tradeoff assessment is independent of the values of each MOE, then the same tradeoff ratio can be applied between the next pair of alternatives to determine their ranking. On the other hand, if this independence does not hold, a new tradeoff assessment must be made. When more than two MOEs comprise the irreducible set of important MOEs, the ranking task complexity increases rapidly. Many more mixed-dominance cases become possible. For any two alternatives, where the dominance is mixed across the various MOEs, a tradeoff assessment among several MOEs must be made. A DM must simultaneously consider the relative importance of three or more MOEs. Even if a DM finds that he can make such a tradeoff assessment among three or four MOEs, he will probably not be able to judge his consistency between his consideration of successive pairs of alternatives, or between his assessments at two different times. At four or more MOEs we can conclude that even the initial tradeoff assessment between one pair of alternatives will be an impossible task for the vast majority of DMs.

#### A. The Basic Concept of the Method

As indicated by this brief discussion, a DM faced with a resource allocation problem with multiple MOEs must make tradeoff assessments among the levels of MOEs achievable by the alternative programs being considered. The cases we are considering are when an irreducible set of two or more MOEs characterize the outputs of the alternative programs. By irreducibility we mean that we have no objective, quantifiable model to relate any two or more MOEs in such a manner that they can be replaced by a single derived MOE. Each MOE is considered an attribute of the expected outcome of the alternatives being considered. Thus, our decision

problem is a multi-attribute problem. The terms attribute and MOE are used synonymously in this report.

In attacking multi-attribute problems, it is useful to distinguish between the two-attribute case and the "more-than-two-attribute" case. By the irreducibility assumption, we can do little to assist the decision maker in his tradeoff assessment task for the two-attribute problem. Since no objective model exists to relate the two attributes, the trade-off assessment must be a subjective assessment based on the decision makers experience and judgment. The DM must, in the final analysis, have an internal model of the relationship of the two attributes to the Navy's ability to carry out its assigned mission. The best we can do to assist the DM in this task is to develop procedures for assisting him in organizing his thoughts on the subject, and for improving his judgmental consistency.

In the more-than-two-attribute case we can do significantly more to assist the DM. The principal idea is to attack the problem of the need to simultaneously assess the overall impact of three or more attributes or MOEs.

The essence of the proposed method is to decompose the multi-attribute assessment problem into a sequence of simpler tradeoff assessment tasks. This is accomplished by constructing a sequence of hypothetical outcomes (each consisting of a set of MOE values). These can be viewed as representing the outcomes of hypothetical ED program alternatives that are assumed feasible. Each hypothetical outcome or program differs from the two sequentially adjacent outcomes in only two MOEs. They are constructed in a manner that eventually links the two real program alternatives. Also, the construction is such that the DM is indifferent between each hypothetical outcome, and between any hypothetical outcome and the outcome for one of the real ED programs.

The concept of construction of the sequence of hypothetical programs is illustrated in Figure III-1. In this figure we show two real programs characterized by four MOEs. These programs are characterized by the values of the MOEs designated  $a_1$ ,  $a_2$ , etc., and  $b_1$ ,  $b_2$ , etc. Thus the two real programs, called A and B, are designated as

			MOE 1	MOE <sub>2</sub>	MOE <sub>3</sub>	MOE <sub>4</sub>	
A	-	(	<sup>a</sup> 1	<sup>a</sup> 2	<sup>a</sup> 3	<sup>a</sup> 4	)
A'	=	(	<sup>b</sup> 1	c <sub>2</sub>	a <sub>3</sub>	<sup>a</sup> 4	)
A''	-	(	<sup>b</sup> 1	<sup>b</sup> 2	c <sub>3</sub>	<sup>a</sup> 4	)
A'''	-	(	<sup>b</sup> 1	b <sub>2</sub>	b <sub>3</sub>	c <sub>4</sub>	)
В	=	(	<sup>b</sup> 1	<b>b</b> <sub>2</sub>	b <sub>3</sub>	b <sub>4</sub>	)

FIGURE III-1 HYPOTHETICAL PROGRAM CONSTRUCTION

$$A = (a_1, a_2, a_3, a_4)$$

and

$$B = (b_1, b_2, b_3, b_4)$$

Three hypothetical programs are designated by A', A'', and A''', and are also shown in Figure III-1. Note that the two attributes in which each successive pair of programs differ are indicated within each box.

Comparing A' to A, we see that A' has the same MOE values except for the first pair of MOEs. The value of  $MOE_1$  for A' is set equal to the value of  $MOE_1$  for B, and the value of  $MOE_2$  is set equal to  $c_2$ . The procedure for obtaining  $c_2$  (and the other  $c_i$ 's) will be described in the following paragraph. In a similar manner A'' has the same MOE values as A' except for the pair of MOEs consisting of  $MOE_2$  and  $MOE_3$ .  $MOE_2$  is set equal to  $b_2$ , the value of  $MOE_2$  for B, and the value of  $MOE_3$  is set equal to  $c_3$ . Through this stepwise procedure we progress from hypothetical alternatives that more closely match A to hypothetical alternatives that more closely match B.

The hypothetical programs in Figure III-1 are obtained by the assignment of values to  $c_2$ ,  $c_3$ , and  $c_4$  by the DM. He chooses these values so that he will be indifferent between any adjacent pair of programs. Thus, the construction of the hypothetical programs does not entail lengthy analysis to establish the values of the MOEs nor does it imply that the program is feasible. Combinations of the MOE values of the real programs provide a basis for their construction, and the choices of MOE values by the DM are made on the basis of assumed feasibility.

After the construction of A''', we see from Figure III-1 that the preference between A''' and B can be determined solely on the basis of the values for  $MOE_4$ . If  $b_4$  is equal to  $c_4$ , the DM is indifferent between A''' and B; if  $b_4$  exceeds  $c_4$ , he prefers B; otherwise, he prefers A'''. Since he is indifferent between A and A''', his preference between A and B is established. In this process, we see that A''' (as well as A' and A'') serves as a surrogate for A.

The primary objective of constructing the hypothetical programs is to relieve the DM of the task of assessing simultaneous tradeoffs among three or more MOEs; it offers him the less complex, though still difficult task of assessing the tradeoffs between only two MOEs. It thus allows him to focus his attention on that part of his internal model of the important overall effectiveness relationships that relates to the two MOEs.

The output of this approach provides:

- A means of simplifying a DM's subjective preference assessments.
- (2) A preference ordering of the alternate key area technology programs.
- (3) Information on the DM's preferences that can be employed to adjust or construct and evaluate new preferred technology programs.
- (4) A means of focusing on significant judgmental assessments and developing a rationale for these assessments.

The remainder of this section will briefly describe the method, its mathematical basis, and the procedures that comprise the approach. This will be done in the context of the decision problem with deterministic

outcomes. That is, we assume that we can estimate with high confidence the effect of the successful completion of a given ED program on each MOE. The decision problem is to select the most preferred ED programs from among competing programs. In Section V we shall then apply the method to a sample deterministic output problem.

#### B. Development of the Methodology

We start by noting that each alternative ED program can be characterized by its expected outcome. This, in turn, consists of the expected achieved levels of each MOE or attribute relating to the ability of the Navy to carry out its assigned mission. Mathematically, an outcome x is defined as a vector whose components consist of the set of relevant MOEs. Thus, the outcome of each ED program can be characterized by

$$x = (x_1, x_2, ..., x_n)$$
 (III-1)

where x, is the i-th MOE variable, and there is a total of n MOEs.

The decision problem is to assess the benefit of all possible sets of outcome values that can be achieved within a given budget, and select the one set of outcomes that maximizes the overall benefit. The program that is expected to achieve these outcomes is then logically judged to be the most desirable or preferred.

Implicit in this decision process is the existence of some model, at least within the decision maker's mind, of the relative contribution of each MOE toward the net benefit to the overall system objectives. This model then allows the DM to trade off amounts of one MOE against another and select one set of expected outcomes that represents the best compromise among the MOEs. If this were a quantifiable model, we would state that some "preference" or "utility" function f exists that assigns a real number (the value of f) to each outcome vector variable x. That is,

$$f(x) = f(x_1, x_2, ..., x_n)$$
 (III-2)

This function is such that if one outcome vector x is preferred to another outcome vector y, then f(x) is greater than f(y), and if neither is preferred over the other, then f(x) equals f(y).

The forms of the preference function depends on the subjective, but informed, preferences of the DM. If the form of this function and the quantitative assessment of its parameters could be established for a given DM, the decision process could be conveniently decomposed into two primary functions: (1) assessing and updating the DM's preference model, and (2) formulating and selecting ED programs that maximize the DM's preference function. Under this decomposition, the DM need never explicitly make an allocation decision. Rather, he would devote his efforts to quantifying and updating, as circumstances and preferences change, his preference parameters that his staff uses to formulate and select ED programs. Thus, decision making would be formalized and decentralized, with control still remaining with the DM through his preference information inputs.

A significant problem in employing this approach for ED resource allocation problems is the feasibility of accurately assessing the DM's preference function. In the most general case, where we have little information concerning the form of the preference function, we must ask the DM to quantify his preferences among alternative outcomes characterized by more than two attributes. The type of information required could be obtained by sequentially asking the DM to assign a number of preference or utility units characterizing his intensity of preference between sequential pairs of alternative outcomes. A sufficient number of alternative outcomes would have to be sampled to cover the range of possible outcomes of interest. With a sufficient sample of points, the preference function may be usefully approximated by this procedure. The feasibility of this approach is questionable because a valid response to the above question may be very difficult or impossible for the DM.

Due to this consideration, other approaches to preference assessments break the task into a number of smaller tasks, and employ some assumption regarding the functional form of the preference function. One particularly convenient form is the separable form given by

$$f(x) = f[f_1(x_1), f_2(x_2), ..., f_n(x_n)]$$
 (III-3)

where each of the f<sub>i</sub>'s is a function of only one variable x<sub>1</sub>.

The validity of using the separable form can be checked by verifying sets of assumptions about the DM's preferences. Two basic assumptions are referred to as preferential independence and utility independence.\* A pair of attributes  $(x_1, x_2)$  is preferentially independent of the attributes  $(x_3, \ldots, x_n)$  if preferences among  $(x_1, x_2)$  pairs, given that  $(x_3, \ldots, x_n)$  are held fixed, do not depend on the level where  $(x_3, \ldots, x_n)$  are fixed. Preferential independence implies that the tradeoffs between attributes  $x_1$  and  $x_2$  do not depend on  $x_3, \ldots, x_n$ . The attribute  $x_1$  is utility-independent of the other attributes  $(x_2, \ldots, x_n)$  if preferences among lotteries over  $x_1$  specifying various amounts of  $x_1$  and probabilities of receiving them, given that  $x_2, \ldots, x_n$  are fixed, do not depend on the levels where they are fixed.

When the above form is valid, the preference assessment task requires determining the  $\mathbf{f_i}$  functions corresponding to each attribute. Procedures for determining the  $\mathbf{f_i}$ 's have been developed  $^\ddagger$  employing tradeoff assessments between pairs of attribute levels.

Assessment of preferences using the above model and approach is difficult but feasible in some situations. It is likely that several iterations of such assessments will be required before the DM is comfortable with the results. The advantage of this approach is that the tasks have been broken down to the point that the DM focuses on one attribute at a time to assess the form of the  $f_i$ 's, and on pairs of attributes to appropriately scale the relative magnitudes of these functions. A serious

D. V. Winterfeldt et al., "Multi-Attribute Utility Theory: Models and Assessment Procedures," AD-770 576, Technical Report, Contract N00014-67-A-0181-0049, University of Michigan, Ann Arbor, MI (5 November 1973).

<sup>&</sup>lt;sup>T</sup>A lottery over  $x_1$  is a chance to increase  $x_1$  by a given amount with probability p, versus a chance to decrease  $x_1$  by some other amount with probability 1 - p.

P. C. Fishburn, "Methods for Estimating Additive Utilities," Management Science, Vol. 13, pp. 435-453 (1967).

weakness in the above model is that it is based on the assumptions of preferential and utility independence. When these assumptions do not hold,\* the procedure is no longer valid.

The approach advocated in this report does not require these restrictive independence assumptions. However, when this independence does hold, the approach can exploit this fact. The approach requires only tradeoff assessments between pairs of attributes. These assessments are made at points in the outcome space related to and including the feasible alternatives being evaluated. Thus, there is a closer coupling between the DM's preference considerations and the set of feasible outcomes achievable from the set of alternative ED programs.

The proposed approach starts with a set of alternative ED programs, and the corresponding achievable outcomes. These are used to establish points in the outcome space at which preference assessments are made. For each pair of achievable outcomes (characterized by n attributes, which may all differ from each other in achievable level), a sequence of hypothetical outcomes are generated that link the pair of outcomes in the following manner. Each hypothetical outcome differs from the previous hypothetical outcome in the level of only two out of n attributes. The first hypothetical outcome differs in this manner from the first real outcome, and the last hypothetical outcome differs similarly from the second real outcome. Thus, if there are n attributes, n - 1 hypothetical outcomes are required to link any two real outcomes. Preference assessments are then made by the DM at each of these outcome points.

It is important to observe that although we have apparently expanded our problem of comparing two n-dimensional real outcomes to comparing n n-dimensional outcomes, in reality we have effectively decomposed the comparison to comparing n 2-dimensional outcomes. This is a consequence of the fact that each outcome in the sequence differs from the previous in only two attributes. Since all other attributes have identical levels, quantifying tradeoff assessments between outcomes only involves thinking about two attributes at a time, a 2-dimensional problem.

A linear preference function is one case where these independence assumptions do hold.

The last statement above is not completely accurate since the levels of the remaining n-2 attributes cannot be simply discarded and not considered in the tradeoff assessment. The DM must firmly establish in his mind what the levels of the n-2 attributes are and their implications for the tradeoffs of the remaining 2 attributes. He is really making a conditional tradeoff assessment, conditioned on the levels of the remaining attributes. If it turns out that the levels of the remaining attributes do not affect his tradeoff assessment, then the preferential independence assumption stated above is satisfied and this fact can be exploited to reduce subsequent tradeoff assessments between other outcome pairs.

After tradeoff evaluations between a pair of outcomes is accomplished, a second pair of outcomes consisting of the preferred outcome from the previous set and one of the remaining outcomes from the feasible set are then selected. The procedure used to compare the previous set of outcomes can then be repeated to establish the next preferred outcome. After all outcomes have been evaluated in this manner, we are left with the most preferred outcome. The ED program that achieves the most preferred outcome is the most preferred program.

This approach will be illustrated with an example. Initially we will consider a very simple example and develop the necessary procedures to implement the approach, from which certain implications can be drawn that lead to more efficient procedures.

Consider simple situation in which there are three important MOEs that measure the benefits of any ED program. Further assume that these MOEs are quantifiable and vary depending on the manner in which funds are expended on exploratory development. Each alternative way of spending a fixed budget is a unique ED program. The outcome of each ED program can be specified in terms of the MOEs or attributes. If there are two alternative programs A and B, and the relevant outcome attributes are designated  $x_1$ ,  $x_2$ , and  $x_3$ , respectively, we can say that program A will result in  $x_1 = a_1$ ,  $x_2 = a_2$ , and  $x_3 = a_3$ , and program B will result in  $x_1 = b_1$ ,  $x_2 = b_2$ , and  $x_3 = b_3$ . We associate these outcomes with programs A and B, respectively, and designate the two outcomes:

$$A = (a_1, a_2, a_3)$$
 (III-4)

and

$$B = (b_1, b_2, b_3)$$
 (III-5)

We also assume that the  $x_i$ 's are defined so that, all else being equal, a higher level of  $x_1$  is preferred to a lower level. This is not restrictive because we can always transform the MOE so that this will be true. Thus, if  $b_i$  was greater than  $a_i$  for i=1, 2, and 3, then we would prefer B to A and would fund program B. On the other hand, if  $b_2$  and  $b_3$  were greater than  $a_2$  and  $a_3$ , respectively, while  $a_1$  was greater than  $b_1$ , it would not be clear which was more preferred. It would depend not only on how much  $a_1$  was greater than  $b_1$  and how much  $b_2$  and  $b_3$  were greater than  $a_2$  and  $a_3$ , but also on the relative importance of the attribute  $x_1$  compared to  $x_2$  and  $x_3$ .

In comparing these two outcomes, the difficulty is in simultaneously trading off three different attributes. The problem is, of course, more severe if there are more than three attributes. To decompose this problem so that a tradeoff between only two attributes is necessary, consider a third hypothetical outcome--call it A'. Let A' have the following values:

$$A' = (b_1, c_2, a_3)$$
 (III-6)

where c<sub>2</sub> is to be determine.

At this point we introduce a tableau in Figure III-2 that will help in visualizing the following discussion, and will form the primary tool in the proposed procedure. In Figure III-2 each row corresponds to an alternative (real or hypothetical) and each column corresponds to an MOE. The vertical dashed line separates those MOEs for which A dominates B from those for B dominates A. The box indicate the responses required from the DM.

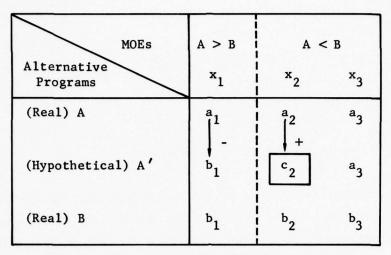


FIGURE III-2 TRADEOFF ASSESSMENT TABLEAU

To determine  $c_2$  we ask the DM to consider outcomes A' and A, and to firmly establish in his mind the values of  $a_1$ ,  $a_2$ , and  $a_3$ , and the significance of the corresponding attributes  $x_1$ ,  $x_2$ , and  $x_3$ . We then ask him to fix  $a_3$  in his mind and consider variations in the levels of  $x_1$  and  $x_2$  from the values  $a_1$  and  $a_2$ . In particular, if  $x_1$  were decreased from  $a_1$  to  $b_1$ , and  $x_2$  could increased from  $a_2$  to compensate, how much would  $x_2$  have to be increased to make the DM indifferent to achieving outcome  $(a_1, a_2, a_3)$  or outcome  $(b_1, a_2 + \Delta, a_3)$  where  $\Delta$  is the amount of his response? The value of  $c_2$  is then set equal to  $a_2 + \Delta$ , and we conclude that he is indifferent to obtaining outcome A or A'. We can write

$$A = (a_1, a_2, a_3) \sim (b_1, c_2, a_3) = A'$$
 (III-7)

where the symbol  $\sim$  means indifference (the symbols > and < will be used to indicate "preferred to," and "less preferred than," respectively).

In Figure III-2, the pair of arrows with the + or - sign indicate the pair being traded off, and the sign of the changes required.

We can now compare  $b_2$  to  $c_2$ . If  $b_2$  is greater than or equal to  $c_2$ , and remembering that  $b_3$  is greater than  $a_3$ , we can infer that B is preferred to A', which is equivalent to A. That is,  $B > A' \sim A$ . This implies that B is preferred to A and we have accomplished our objective of ranking A and B. On the other hand, if  $b_2$  is less than  $c_2$  we do not know yet whether A is preferred to B or vice versa.

In the latter circumstance, we consider a new hypothetical outcome  $A^{\prime\prime}$  having the following values:

$$A'' = (b_1, b_2, c_3)$$
 (III-8)

where  $c_3$  is to be determined. Figure III-3 shows an expansion of the tableau from Figure III-2 to include A''. To determine  $c_3$  we ask the DM to consider outcomes A' and A'', and to consider variations in the levels of  $x_2$  and  $x_3$  from values  $c_2$  and  $a_3$ . We ask, if  $x_2$  were decreased from  $c_2$  to  $b_2$ , how much would  $a_3$  have to be increased to make him indifferent to  $(b_1, c_2, a_3)$  or  $(b_1, b_2, a_3 + \Delta)$ ? Again  $c_3$  is set equal to  $a_3 + \Delta$  and we can write

$$A \sim A' = (b_1, c_2, a_3) \sim (b_1, b_2, c_3) = A''$$
 . (III-9)

MOEs	A > B	A < B			
Alternative Programs	× <sub>1</sub>	× <sub>2</sub>	× <sub>3</sub>		
(Real) A	a <sub>1</sub> -	a <sub>2</sub> +	<sup>a</sup> 3		
(Hypothetical) A'	ь́ <sub>1</sub>	c <sub>2</sub>   -	a <sub>3</sub> +		
(Hypothetical) A''	<sup>b</sup> 1	b <sub>2</sub>	[c <sub>3</sub> ]		
(Real) B	<sup>b</sup> 1	ь <sub>2</sub>	<sup>b</sup> 3		

FIGURE III-3 EXPANDED TRADEOFF ASSESSMENT TABLEAU

We now compare  $b_3$  to  $c_3$ . If  $b_3$  is greater than  $c_3$ , the DM prefers B to A''. If  $b_3$  equals  $c_3$ , then he is indifferent between B and A''. Finally, if  $b_3$  is less than  $c_3$ , he prefers A'' to B. In each of the three eventualities, A'' (as well as A') is a surrogate for A since A  $\sim$  A'  $\sim$  A''.

Thus, whatever the preference relation between B and A'', the same preference holds between B and A.

Now consider a brief numerical example. Let

$$A = (10, 5, 7)$$
 (III-10)

and

$$B = (7, 7, 9)$$
 . (III-11)

We construct the tableau shown in Figure III-4. The question is how much must  $\mathbf{x}_2$  be increased to compensate for a 3-unit decrease in  $\mathbf{x}$ , from 10 to 7 with all other MOEs equal? If the response is to increase  $\mathbf{x}_2$  from 5 to 6 we update the tableau to Figure III-5. Comparing B to A' in each MOE, we see that B is not dominated by A' in any MOE, and that it dominates A' in at least one MOE (two in this case,  $\mathbf{x}_2$  and  $\mathbf{x}_3$ ). Thus,  $\mathbf{B} > \mathbf{A}' \sim \mathbf{A}$  or  $\mathbf{B} > \mathbf{A}$ .

MOEs	A > B	A < B			
Alternative Programs	×1	× <sub>2</sub>	× <sub>3</sub>		
(Real) A	10	5	7		
(Hypothetical) A'	7	?	7		
(Hypothetical) A''	7	7			
(Real) B	7	7	9		

FIGURE III-4 NUMERICAL EXAMPLE TABLEAU

Now assume instead that the response was to increase  $\mathbf{x}_2$  from 5 to 8. This result is shown in Figure III-6. Since A' dominates B in  $\mathbf{x}_2$  and is dominated by B in  $\mathbf{x}_3$ , we cannot yet rank A and B. A second tradeoff

assessment is required and the question is how much must  $x_3$  be increased to compensate for a decrease of  $x_2$  from 8 to 7 with all other MOEs equal? If the response is to increase  $x_3$  from 7 to 8, we conclude that  $B > A'' \sim A' \sim A$ , or B > A. On the other hand, if  $x_3$  were increased to 10, the opposite would be true and A > B.

MOEs	A > B	Α -	< B
Alternative Programs	× <sub>1</sub>	× <sub>2</sub>	<b>x</b> 3
(Real) A	10	5+	7
(Hypothetical) A'	7	6	7
(Hypothetical) A''	7	7	
(Real) B	7	7	9

FIGURE III-5 TABLEAU UPDATE--CASE 1

MOEs	A > B	A	< B
Alternative Programs	×1	*2	*3
(Real) A	10	5	7
(Hypothetical) A'	7	8	7
(Hypothetical) A''	7	7	?
(Real) B	7	7	9

FIGURE III-6 TABLEAU UPDATE--CASE 2

We can now readily see how to generalize this procedure to the case of n MOEs. We first select two alternatives, A and B, and inspect the successive pairs of values for each MOE. We find that in m cases A > B, in p cases  $A \sim B$ , and in q cases A < B, where m + p + q = n. We select the minimum of m and q (assume it is m) and rearrange the MOEs so that the first m consist of the case where A > B, the next q consist of the case where A < B. For specificity assume that n = 6, m = 2, q = 3, and p = 1. We can now construct the tableau shown in Figure III-7.

Figure III-7 shows several interesting properties of this procedure. First, the maximum number of hypothetical alternatives, and thus the

MOEs	A	> B		B > A		A ~ B
Alternative Programs	× <sub>1</sub>	*2	× <sub>3</sub>	<b>x</b> <sub>4</sub>	× <sub>5</sub>	×6
(Real) A	<sup>a</sup> 1	a <sub>2</sub> +	a <sub>3</sub>	a <sub>4</sub>	<sup>a</sup> 5	a <sub>6</sub>
(Hypothetical) A'	b <sub>1</sub>	c <sub>2</sub>	a <sub>3</sub>	<sup>a</sup> 4	<sup>a</sup> 5	<sup>a</sup> 6
(Hypothetical) A''	<sup>b</sup> 1	b <sub>2</sub>	c <sub>3</sub>	a <sub>4</sub>	<sup>a</sup> 5	a <sub>6</sub>
(Hypothetical) A'''	<sup>b</sup> 1	b <sub>2</sub>	b <sub>3</sub>	c <sub>4</sub>	a <sub>5</sub>	<sup>a</sup> 6
(Hypothetical) A''''	<sup>b</sup> 1	<sup>b</sup> 2	ь <sub>3</sub>	-   b <sub>4</sub>	c <sub>5</sub>	<sup>a</sup> 6
(Real) B	<sup>b</sup> 1	b <sub>2</sub>	ь <sub>3</sub>	b <sub>4</sub>	ь <sub>5</sub>	b <sub>6</sub> = a <sub>6</sub>

FIGURE III-7 TRADEOFF ASSESSMENT TABLEAU--6-MOE CASE

maximum number of tradeoff assessments, is m+q-1 or, in this case, 2+3-1=4. The minimum number of tradeoff assessments required is  $\max(1, m-1)$ , or in this case,  $\max(1, 2-1)=1$ . (Note that at least one tradeoff assessment will always be required.) This minimum number of tradeoff assessments would occur if  $c_2$  were less than or equal to  $b_2$  (or in general if  $c_m$  were less than or equal to  $b_m$ ). In such a case, B would dominate the (m-1)-th hypothetical alternative in at least q-3 MOEs and not be dominated by the (m-1)-th hypothetical alternative in any MOE. Thus, B would be preferred to A.

If, in fact, B > A we discover that fact anywhere from the (m-1)-th tradeoff assessment to the (m+q-1)-th tradeoff assessment. If, on the other hand, A > B or  $A \sim B$  we would discover that fact only after the (m+q-1)-th tradeoff assessment.

In Figure III-7 we can also note the relationship between any hypothetical alternative and each of the pair of real alternatives. We see that the first k MOEs of the k-th hypothetical alternative are equal in value to the MOEs of alternative B, and the last n-k-1 MOEs are equal to the MOEs of alternative A. Thus, the hypothetical alternatives can readily be constructed from alternatives A and B. The remaining MOE value to complete each hypothetical alternative is supplied by the tradeoff assessment of the DM. He accomplishes this without any thought as to whether or not the hypothetical alternative is feasible or what the real cost may be. This is true since even if the alternative were feasible, he need never seriously consider implementing it. If it costs more than or equal to the budget, he will select either A or B, since A is as good and B may be better and neither costs more. If it costs less than the budget, he should advise his staff to find an improve alternative whose cost equals the budget, and he will select either the new alternative or B.

At this point in the development we present a mathematical and graphical model of the tradeoff assessment process outlined above, and use it to derive one additional useful concept.

In our three MOE examples let the preference function be represented by

$$f(x) = f(x_1, x_2, x_3)$$
 (III-12)

A 4-dimensional graph is needed to plot any specific form of Eq. (III-12). However, we can ease that task by fixing one of the values of  $\mathbf{x}_i$  and plotting contours of the function in two dimensions. Thus, let  $\mathbf{x}_3$  be equal to 7, the value of  $\mathbf{a}_3$  in the previous numerical example. We can now plot the locus of all combinations of  $\mathbf{x}_1$  and  $\mathbf{x}_2$  that correspond to a fixed value of  $\mathbf{f}(\mathbf{x})$  when  $\mathbf{x}_3$  is held at 7. Figure III-8 shows several of these loci for various values of  $\mathbf{f}(\mathbf{x})$ . These contours are iso-preference curves

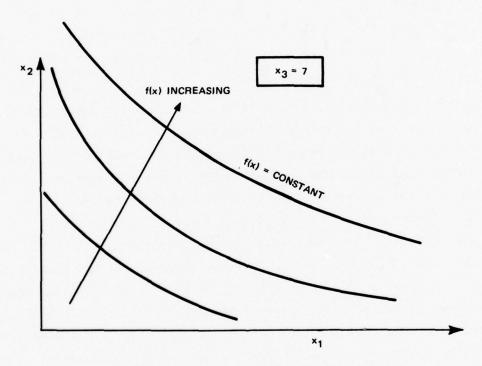


FIGURE III-8 ISO-PREFERENCE CONTOUR GRAPH

so that the DM is indifferent between any two points on the same curve. In selecting the shape of these contours we have assumed that as one MOE gets smaller, an increasing amount of change is required to compensate. This is not a vital assumption, but would apply for many types of MOEs.

We can view these iso-preference contours as contours of a hill rising out of the plane of the paper. Higher values of f(x) correspond to

higher altitude on the hill, and we prefer to allocate our budget on a program with outcome x that has the largest value of f(x). Thus, we wish to climb the hill, but our budget constrains how high we can climb as a function of the direction we climb. Figure III-9 shows a budget constraint curve superimposed on the iso-preference curves. Also shown in Figure III-9 are the points A and A' from the previous numerical example, and

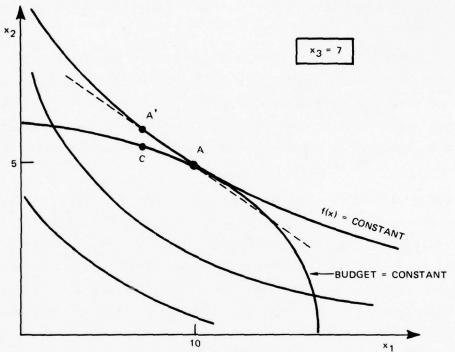


FIGURE III-9 ISO-PREFERENCE CONTOUR GRAPH WITH BUDGET CONSTRAINT

another point C that is a feasible outcome and alternative within the same budget as A. A' is the hypothetical alternative constructed so that it lies on the same iso-preference curve, but as shown is not feasible within the budget constraint. Point C has more of MOE  $\mathbf{x}_2$  but less of MOE  $\mathbf{x}_1$  compared to point A. Since C and A' differ in only one MOE, and since A' dominates C in that MOE, we conclude that A  $\sim$  A' > C.

Returning to the hill-climbing analogy, the budget constraint curve is like a fence constructed on the hill. Our objective is to climb as high as possible without crossing the fence. As shown, point A is the highest we can get and represents a preferred alternative to point C.

In assessing the DM's tradeoffs between  $x_1$  and  $x_2$  we have effectively obtained the slope of the line through points A and A' (the full forms of the iso-preference curves are unknown to us). The slope represents a local approximation to the iso-preference curve through A and A', at least in the region of A, A', and C. We can then conclude that we are likely to find preferred points above this tradeoff line and less preferred points below. As we see in Figure III-9, this relationship is true between points A and C.

Of course we have not brought alternative B into the picture as yet, because  $x_3 \neq 7$  for B. To bring in B we must employ a 3-dimensional perspective plot. In this plot, the budget constraint curve will be a curved surface roughly like an ellipsoid, and the iso-preference contours will be like the layers of an onion, some of which intersect the budget constraint surface.

Figure III-10 is an attempt to depict this geometry. In Figure III-10, the long-dashed curves represent the intersections of the budget constrained surface with the  $(x_1, x_2)$ ,  $(x_2, x_3)$ , and  $(x_1, x_3)$  planes. In addition, we show two slices of this surface, where curves  $P_1P_4$  and  $P_2P_3$  indicates the intersection of the surface with planes parallel to the  $(x_1, x_2)$  plane and  $(x_2, x_3)$  plane, respectively. These slices were selected to pass through points (or alternatives) A and B, respectively. The budget constrained surface is convex to the viewer.

Of course there are an infinite number of iso-preference surfaces in this 3-dimensional space. These surfaces are all concave to the viewer. Figure III-10 shows a portion of the surface passing through point A. This portion is defined by curves AA', A'A'', and A''A. Note that curve AA' is in the plane of the slice through point A, and curve A'A'' is in the plane of the slice through point B. Curve A''A is in a plane parallel to the  $\mathbf{x}_3$  axis. Also note that the iso-preference surface depicted intersects the budget constrained surface at point A, and at the two points where curves AA', and A''A transition to dashed lines. Point B meanwhile is below the iso-preference surface, implying that A > B. The purpose of the method is to determine this fact without full knowledge of the shape and location of the iso-preference surface.

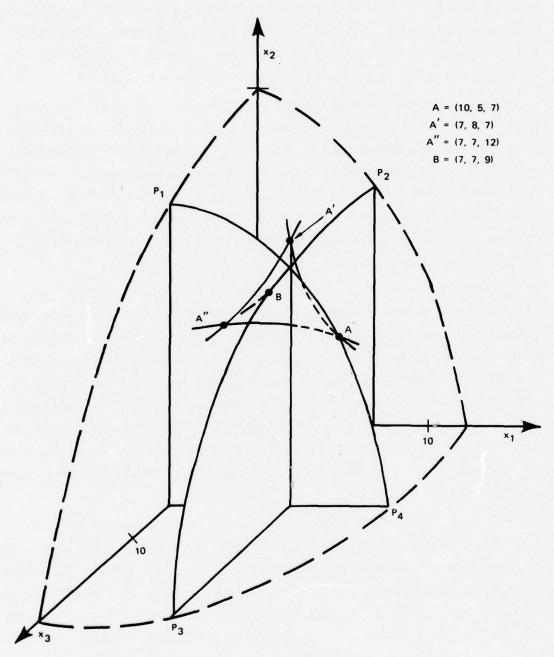


FIGURE III-10 THREE-DIMENSIONAL PERSPECTIVE GRAPH

In the previous example we accomplish this by determining the locations of points A' and A'' from the DM's tradeoff assessments. In this particular case we assume that the DM's responses are such that A'' dominates B in the  $\mathbf{x}_3$  MOE while it equals B in the other MOEs. Thus, whatever the shape of the iso-preference surface, B must lie below that surface, and A, which is on the surface, must be preferred to B.

Just as in the two-dimensional case where we concluded that points A and A' defined a line approximating the two-dimensional iso-preference curve, we can see that points A, A', and A'' define a plan that locally approximates the iso-preference surface in the three-dimensional case. We also note that point B is below this plane. Furthermore, we can extrapolate to the n-dimensional case and conclude that the hypothetical alternatives define an (n-1)-dimensional hyperplane approximating the iso-preference hypersurface.

Returning to the 3-dimensional case, we can note that there are regions of the budget constrained surfaced that lie above the iso-preference surface shown in Figure III-10. The alternative programs represented by points in this region are preferred to A. In the case illustrated we have assumed that the outcome MOEs of each feasible and budget constrained alternative are continuously variable. Thus, there will be a most preferred alternative other than A. In practice the MOEs may not vary continuously and the budget constrained surface may reduce to a finite number of separate points, one of which may or may not lie above the iso-preference curve through A.

Since we do not know the shape of the iso-preference curve we can only consider what information we do have about the iso-preference curve. This information consists of the fact that A, A', and A'' all lie on the same preference surface, and they define a plane that is a local approximation to that surface. Points on the budget constrained surface may or may not lie above this plane. However, if a budget constrained point does lie above this plane, it will likely also lie above the iso-preference surface. Based on this state of information we can conclude that the budget constrained point or alternative that is furthest above the plane will likely be the most preferred. We can readily find that point--say

it is C--and determine the relative ranking of A and C through the application of the tradeoff method.

In case there is no budget-constrained point above the tradeoff plane, we can find the point below the plane that is closest to the plane. This point represents the most likely point, from among those remaining, that may be preferred to A.

Unfortunately, whichever is preferred, the hyperplane concept does not necessarily select the most preferred from the set of candidates. This is because, without restrictive conditions on the shape of the isopreference surface, C was chosen based only on the approximation represented by the plane through A, A', and A''. Thus, we must still apply the tradeoff methodology to points A and C to determine conclusively which is preferred.

The procedure for selecting C will at the minimum lead to maximizing the probability that we will have considered and ranked the most preferred alternative early on. At the maximum, the tradeoff assessment process and data could sharpen the DM's judgmental perspective to the point where he can assess the adequacy of the representation of his preference tradeoffs by the information encoded in the most recent hyperplane. This information represents a linear model of the DM preference tradeoffs. If this linear model is considered a valid representation of his preference tradeoffs over the region of the remaining alternatives, it can be used to rank these remaining alternatives without further tradeoff assessments. No decision rules have been found to ensure that this last procedure would obtain the most preferred alternative. Thus, it can only be included as an optional procedure based on the DM's subjective judgment.

The procedure for selecting successive alternatives for the tradeoff method consists of finding the feasible budget constrained point from among those not previously ranked that maximizes the linear function,

$$g(x) = \sum_{i=1}^{n} v_{ni} x_{i}$$
 (III-13)

This function, g(x), measures the "distance" of any vector  $(x_1, x_2, \ldots, x_n)$  above the tradeoff hyperplane characterized by the vector  $(\gamma_{ni}, \gamma_{n2}, \ldots, \gamma_{nn})$ . The  $x_i$ 's are the MOE values for each feasible alternative, and the  $\gamma_{ni}$ 's are the set of slopes defining the hyperplane. For i=1 to n, each  $\gamma_{ni}$  is the slope of the hyperplane in the  $(x_n, x_i)$ -th plane. This implies that  $\gamma_{nn} = 1$ . The  $\gamma_{ni}$ 's are obtained from the DM's response data after each application of the tradeoff assessment procedure. The derivation of Eq. (III-13) and the definition of the  $\gamma_{ni}$ 's are developed in Appendix A.

# C. The Procedures of the Method

The procedures described in this section comprise a tradeoff method for multi-attribute decision problems. The method is applied after appropriate MOEs have been derived and budget constrained feasible ED alternatives have been constructed and described in terms of the MOEs. It is also assumed that there is no alternative that dominates all other alternatives in each MOE, and that all alternatives that are dominated in each MOE by all other alternatives have been discarded. In the former case, there is no need for the method and the dominant ED alternative should be funded.

In the procedures below, the word "dominated" is used in the preferential sense rather than the quantitative sense. Thus, if a smaller value of a particular MOE is preferred to a larger value, smaller numerical MOE values will dominate larger numerical MOE values.

The tradeoff assessment method consists of the following steps:

- (1) Select alternative pair. Initially, select the two potentially most preferred alternatives based on the a-priori subjective assessment of the DM. On subsequent iterations through Step 1, select the current highest-ranking alternative and one other potentially most preferred alternative from among the remaining alternatives.
- (2) Reorder MOEs. Reorder the MOEs so that the minimum number of dominated MOEs between the two alternatives are at the beginning of the sequence of MOEs, and the remaining reverse-dominated MOEs are listed next followed by the remaining equal-value MOEs (if any).

- (3) Construct tableau. Construct the tradeoff assessment methodology tableau so that the top alternative dominates the bottom alternative in the first MOE.
- (4) Perform tradeoff assessment. Obtain the DM's response between the appropriate pair of MOEs within the tableau. Each response completes the construction of a hypothetical alternative.
- Test for dominance. After the minimum number of tradeoff assessments (m - 1) have been completed, determine whether the bottom alternative completely dominates the last hypothetical alternative. If it does, the bottom alternative is ranked as more preferred than the top alternative. If unranked alternatives remain, we proceed to Step 1 or Step 6 at the option of the DM. Otherwise, we have completed the procedure and the most preferred alternative has been identified. If, on the other hand, dominance has not yet occurred, further tradeoff assessments are required and we return to Step 4. After the maximum number of tradeoff assessments (m + q - 1) have been completed, the two alternatives are ranked by comparing the last MOE involved in the tradeoff assessment. Again, we proceed to Step 1 or Step 6 if unranked alternatives remain.
- (6) Complete the tradeoff assessments (optional). If a complete set of tradeoff ratios have not been established, the Yni's in Eq. (III-13) are not all known, and the linear function cannot be optimized to determine the next alternative. However, at the option of the DM, the remaining tradeoff assessments (obtained according to Step 4) can be accomplished. The Yni's, which are tradeoff ratios between the n-th or last MOE and the i-th MOE can then be computed according to Eq. (A-12) in Appendix A.
- (7) Perform linear optimization for next alternative selection. Select the next alternative by optimizing Eq. (III-13) over all remaining alternatives.
- (8) Test for termination (optional). Ask the DM to carefully consider each of his most recent tradeoff ratios and the range of MOE values covered by the remaining alternatives. Determine whether he would modify any of these tradeoff ratios as a function of the MOE values within this range. If he would not, the choice must be made between the alternative determined in Step 7 and the current highest-ranking alternative. Of these two, the preferred one is the alternative which optimizes the linear function--Eq. (III-13). At this point, the most preferred alternative has been identified and we are done. If the DM indicates that his tradeoff ratios are not constant over the range of MOE values, proceed to Step 2 with the current highest-ranking alternative and the alternative obtained in Step 7.

### IV DEVELOPMENT OF MEASURES OF EFFECTIVENESS FOR LOGISTICS

### A. General

As discussed earlier, one of the three objectives of this research was to identify and develop measures of effectiveness (MOE) models for one key logistics technical area. We decided to focus on one technical area due to the complexity of the naval logistics system, and the interactions among the various technical areas. By selecting this course, we could arrive at the point of describing alternative ED programs in terms of their multiattribute structure much earlier. We could then construct a specific example, and determine how the proposed resource allocation method would be applied. This would allow us to evaluate the usefulness of the method, and the course to pursue to apply it to other logistics technical areas.

The Logistic Supply System Segment of the Technical Strategy for Logistics and Facilities was chosen for this purpose after several discussions with the sponsor of this research. In this segment of the strategy, a number of projects were currently under way, and the consequences of supplemental funding alternatives would be of more than academic interest. In addition, a number of new starts were also being considered. These projects are shown in Table IV-1 together with test case funding levels, and conjectured supplemental or new funding amounts. These projects are designated C1 through C10, respectively.

In Table IV-2, the conjectured present funding and postulated add-on funding structure is illustrated to form the basis for developing alternative ED programs. Note in Table IV-2 that projects C2 and C8 have been deleted from the list. C2 was dropped because one of the primary MOEs identified--responsiveness--did not apply to this project, and in any case no new add-ons were being considered. Thus, inclusion of C2 would require the introduction of new MOEs but would not affect any possible ED funding decisions. Other projects in the list also would not affect funding decisions (i.e., C1, C3, and C6), but these were retained because

Table IV-1
SUPPLY SYSTEM PROJECTS

Project Number	Project	Cost (thousands of dollars)
C1	Vehicle Scheduler	\$ 120
C2	Transportation of Personal Effects	500
С3	Material Distribution System	280
C4	Container Network Analysis	100 (50)*
C5	Information and Material Movement System	210 (200)
C6	Warehousing	130
C7	Parcel Handling	80 (200)
	New Starts	
C8	Resource Allocation Methodology	(200)
С9	Multi-Echelon Repairables	(300)
C10	Interwarehouse Transport	(300)

 $<sup>^{*}</sup>$  Numbers in parentheses represent add-on funding.

Table IV-2
SUPPLY SYSTEM PROJECT FUNDING
(Thousands of Dollars)

Present Funding		
Project C1	\$120	
Project C3	280	
Project C4	100	
Project C5	210	
Project C6	130	
Project C7	80	
Project C9		Note: C2 and C8 omitted
Project C10		
	\$ 920	
Total Postulated	Add-ons	
Project C4	\$ 50	Note: C4, C5, C7 are
Project C5	200	add-ons to already committed
Project C7	200	programs. C9, C10 are new programs.
Project C9	300	L. S. L.
Project C10	300	
	\$1050	
Total, with add-ons	\$1970	
		Add-on 25% - \$1182 (\$262)
Intermediate Inc	rements	<u> </u>
		75% - 1707 (787)

their MOEs were consistent with the other projects. C8 represents funds that would be allocated to pursue this type of resource allocation method study. The principal MOE for this type of project would be related to dollars saved on misdirected projects. This, of course, would be very difficult to estimate, and whatever one might estimate would be subject to an uncertainty much out of line with what outcomes one might estimate from the remaining projects. For these reasons and with the consent of the sponsor, projects C2 and C8 were deleted for the purpose of this research.

From Table IV-2 we see that the present funding level is \$920K with postulated add-ons of \$1050K. We decided to consider three intermediate levels of postulated add-ons. At the 25%, 50%, and 75% level, these correspond to \$262K, \$525K, and \$787K add-on budgets. With these incremental budget amounts, a number of alternative ways of funding can be formed. These are shown in Table IV-3. Note that the funding level for each alternative at each budget level only approximates the three dollar values given above. Table IV-3 shows that at the 25% increment budget we can construct four alternative programs which consist of funding addons in C4 and C5, or C4 and C7, or C9, or C10. (The funding levels do not exactly equal the 25% increment amount, but approximate it as shown.) For the 50% increment budget, we have five alternative programs, and finally for the 75% increment budget we have four alternative programs.

For each incremental budget level, we must now ask which of the program alternatives is most preferred by the DM? To answer this question, the following steps must be performed. First, the important decision MOEs related to the outcomes of the alternative programs must be identified and modeled in terms of lower-order MOEs where applicable. Next, the DM employs some method to assess his preferences and tradeoffs among the MOEs, and to select the most preferred program. The DM can then integrate the information generated in the previous step to develop his rationale for his choice.

Through the course of several discussions with the sponsor, the following common view of the MOE identification and selection process

Table IV-3

INCREMENTAL BUDGET ED PROGRAMS
(Thousands of Dollars)

25% Increment Pr	ograms								
Alternative	I	1	I	I.	II	I	V		
Project C4: C5:	\$ 50 200	C4: C7:	\$ 50 200	C9:	\$300	C10:	\$300		
Total	\$250		\$250		\$300		\$300		
50% Increment Pr	coarame								
John Therement II	Ograms								
Alternative	I	I.	I	I.	II	I	V	V	
Project C4: Project C5:	\$ 50 200	C4: C5:	\$ 50 200	C4:	\$ 50 200	C4: C7:	\$ 50 200	C9:	\$300 300
Project C9:	300	C10:	300	C9:	300	C10:	300		—
Total	\$550		\$550		\$550		\$550		\$600
75% Increment Pr	ograms								
Alternative	I	13		I	II	I	V		
Project C4:	\$ 50	C4:	\$ 50	C5:	\$200	C7:	\$200		
Project C5:	200	C5:	200	C9:	300	C9:	300		
Project C7:	200	C7:	200	C10:	300	C10:	300		
Project C9:	300	C10:	300						
Tota1	\$750		\$750		\$800		\$800		

was reached. There is a hierarchy of MOEs that measure effectiveness at various echelons of Naval activity. Those MOEs at the highest echelon are seldom useful in identifying specific problems and possible solution approaches. On the other hand, they do indicate that problems exist somewhere in the system, and their values indicate the magnitudes of the problems. Thus, they serve as meaningful measures of the outcomes of various solution approaches, once these solution approaches are formulated and their effects on modifying the values of the MOEs are evaluated.

At lower echelons of activity the MOEs become more and more specific and detailed, and are therefore more directly related to the problems in the systems. Thus, they are useful in formulating solution approaches, but may not be very useful in measuring how well the system problem seen at a higher echelon has been solved.

Decision makers at any given level of activity are of course concerned with both sets of MOEs. The lower-echelon MOEs allow the DM to identify specific problem areas, and guide the formulation of appropriate solutions, while the higher-echelon MOEs allow him to assess how well the system problem would be solved with any given course of action (i.e., ED program). Thus, we will henceforth refer to the lower-echelon MOEs as Project MOEs, and the higher-echelon MOEs as the Strategy MOEs.

Strategy MOEs are of course related to the Project MOEs. The relationships must be identified, understood, and modeled so that we can then compute the effect of each proposed ED program on the Strategy MOEs through their effects on the Project MOEs. If only one Strategy MOE existed, and its dependence on a set of Project MOEs could be determined, the resource allocation decision would involve simply selecting that ED program providing the greatest value of the Strategy MOE. Unfortunately, a number of Strategy MOEs exist, and the resource allocation decision must involve a tradeoff process among the various Strategy MOEs.

We proceeded by making a first cut at identifying MOEs for measuring the outcomes of the various supply system projects listed in Table IV-1. A total of 12 potential MOEs were initially identified. However, after reviewing these, it became evident that this list of MOEs contained a mix of MOE hierarchies. We then modified this list and narrowed it down to an initial set of five Strategy MOEs.

Table IV-4 summarizes this process and shows the relationships among the Strategy MOEs and the eight supply system projects. These Strategy MOEs are spelled out in the paper entitled "Technical Strategy

Table IV-4

# PROJECT VERSUS STRATEGY MOES

 $\mathbf{C}_{\mathbf{I}}$  = Capital investment costs

C<sub>OMN</sub> = Costs in O&MN

M = Total manpower required

 $M_S$  = Skilled manpower required

R = Responsiveness

Project Number	Previously Identified Project MOEs	Strategy MOEs	Rationale
C1	Number of vehicles required	C <sub>I</sub> C <sub>OMN</sub> M	Saves on the costs of vehicles Reduces OMN costs because of fewer vehicles Reduces manpower required
	Even workload		Reduces overtime/manpower costs  Reduces required manpower due to greater efficiency  Reduces delays due to workload saturation
	Equipment utilization efficiency	R	Reduces delays due to idle equipment
	Reduced planning time	M <sub>S</sub> C <sub>OMN</sub> R	Reduces skilled manpower required Reduces manpower costs Improved planning reduces delays
С3	Responsiveness	R	
	Dollars saved in capital investment	c <sub>I</sub>	
C4	Dollars saved in capital investment	cI	
	Responsiveness	R	
C5	Responsiveness	R	
•	Dollars saved in capital investment		
	1-NORS	R	NOR is directly related to time awaiting parts or material, which is measured by responsiveness
C6	Dollars saved in capital investment	cI	
		R	Speeds up acquisition of material from warehouse
C7	Dollars saved in operating costs	COMIN	,

Table IV-4 (Concluded)

Project Number	Previously Identified Project MOEs	Strategy MOEs	Rationale
C7 (Concl.)	Control/visibility	R C <sub>OMN</sub> M	Control reduces delays by locating material or parts Reduces costs due to shipment losses Reduces manpower to locate material
	Responsiveness	R	
С9	Dollars saved in OMN	C <sub>OMN</sub>	
	Floor space and volume in warehousing	c <sub>omn</sub>	Reduces need to build additional warehouses  Reduces cost of operating additional warehouses
	Reduction in inventory	R C <sub>OMN</sub> M	May reduce responsiveness due to lack of item in inventory at user echelon or in the system Reduces dollars tied up in inventory items and maintaining inventory Possible change in manpower required
C10	Manpower	M C <sub>OMN</sub>	Reduces manpower costs
	Responsiveness	R	

for Exploratory Development in Logistics." For the supply system, the need is stated to "improve supply system responsiveness at reduced cost and manpower." Responsiveness is measured in supply cycle time, the elapsed time from when a user generates a request for parts or material to the time he receives the requisitioned items. Cost is broken down into O&MN and capital investment dollars. Finally, manpower is broken down into total manpower and skilled manpower.

Further consideration of responsiveness, as measured by supply cycle time, led to the recognition that supply cycle time was highly dependent on the situation or scenario being considered. For example, when one aims at improving the response time for a repair shop to obtain a small spare part, a reduction in response time of hours may be significant, whereas for the acquisition of a major system component such as a new or overhauled jet engine, one looks for response time reductions on the order of days. Thus, we cannot simply average together these different response times to measure the effect of all improvements in the system.

We concluded that response time should be categorized by several scenarios that bring out the types of response time savings expected from the set of candidate projects. The expected response times in each scenario are then considered as separate MOEs.

Also at this point, with the approval of the sponsor, we decided not to attempt to model the manpower Strategy MOEs. This was done to bound the MOE modeling problem and to focus on modeling the responsiveness and cost MOEs.

The supply scenarios consisted of the five scenarios described in Table IV-5. These scenarios are designated A1, A2, B, C, and D, respectively. Scenario A1 applies to the case of a small parts requisition by a repair shop from an LSP or an NLSP, if the part is not available at the LSP. Scenario A2 is similar in that it applies again to the case of

<sup>\*&</sup>quot;Technical Strategy for Exploratory Development in Logistics," David W. Taylor Naval Ship Research and Development Center, Bethesda, MD (25 February 1977).

Table IV-5

SUPPLY SCENARIOS

	The state of the s		The second name of the second na	The second name of the second na	
	Scenario Al	Scenario A2	Scenario B	Scenario C	Scenario D
	Small Requisition from LSP or NLSP	Repair-at-Sea Requisition from LSP or NLSP	Heavy Equipment Requisition from NLSP	High-Volume Supply Requisition from NLSP	Underway Replenishment (UNREP)
User	Repair shop	Combatant ship	A/C maintenance shop or shipyard	Supply center	Combatant ship
Supplier	Local warehouse/ supply point	Own/other ship supplies	Major supply point	ICP	UNREP ship
Distance	LSP: 0.5 → 1.0 miles NLSP: ≈150	LSP: negligible NLSP: 2 miles	500 to 1500 miles	250-3000 miles	Negligible
Transportation mode	LSP: Van/Truck NLSP: Parcel Post	LSP: hand/dolly NLSP: helo/highline	Truck/rail/air	Truck/rail	Highline
Supply item	Small packaged part or assembly	Small spare part or assembly	System component	Normal supplies in quantity	Fill-hull/ consummables
No. of units of issue	1-10	1-10	1	1-1000 pallets 40P = 8' × 8' × 40' volume	6-200 pallets
Shipment preparation	Pre-packaged/ multi-pack	None/ pre-packaged	Crate or individual container	Container/pallet construction	Netting/pallet construction
Loading/ material handling	Hand/dolly/forklift	Hand/dolly	Heavy loading equipment	Forklift/gantry/ straddle truck	Multifaceted
Substitutability	Moderate	Moderate	None	High	High

a small parts requisition, but onboard a ship at sea. In this case, the LSP is the ship's own supply stocks, and the NLSP is another ship's supply stocks. The second ship is assumed to be in the same task force as the ship requiring the repair part. These two scenarios are quite similar as their designation implies, but they were kept separate because of the difference in urgency between the two cases.

Scenario B is a heavy equipment requisition from an NLSP by an air-craft maintenance shop or a shipyard. Scenario C is a high-volume, normal supply requisition from an NLSP. Finally, Scenario D covers the case of underway replenishment at sea.

Through the process described above we arrived at the following set of seven Strategy MOEs for use in the supply system ED program selection task:

- (1)  $R_{\Lambda 1}$  = Expected response time in Scenario Al
- (2)  $R_{\Delta 2}$  = Expected response time in Scenario A2
- (3)  $R_{\rm p}$  = Expected response time in Scenario B
- (4)  $R_C$  = Expected response time in Scenario C
- (5)  $R_D$  = Expected response time in Scenario D
- (6)  $C_{\tau}$  = Capital investment costs
- (7)  $C_{OMN}$  = Operations and maintenance Navy costs.

The remainder of this section deals with the construction of a responsiveness model to measure the expected response time, and a description of the cost model used to measure the costs.

## B. The Responsiveness Model

A simple expected value model has been developed that relates responsiveness (system response time) to various program level MOEs by means of selected system delay times. Figure IV-1 presents a flowchart depicting the generic supply process used as a basis for the model. Appropriate delay times and branch probabilities are indicated on the flowchart. These variables are defined in Table IV-6.

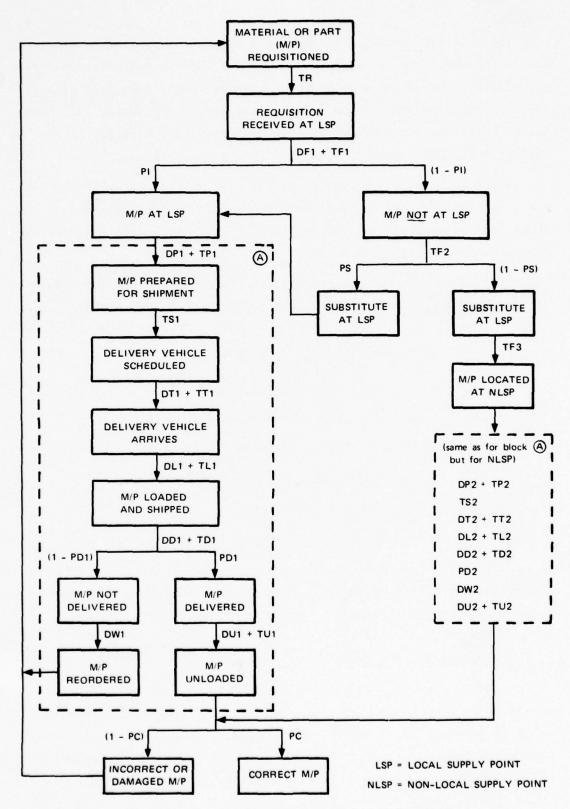


FIGURE IV-1 RESPONSIVENESS MODEL FLOWCHART

### Table IV-6

## RESPONSIVENESS MODEL INPUT PARAMETERS

TR	=	Initial	requisition	processing	time

- DF1 = Delay time prior to material location process
- TF1 = Time to determine whether or not material is at LSP
- TF2 = Time to determine whether or not substitute is at LSP
- TF3 = Time to locate material at NLSP
- DP1 & DP2 = Delay time prior to transportation scheduling/shipment preparation
- TP1 & TP2 = Time to prepare material for shipment
- TS1 & TS2 = Time to schedule vehicle for transportation
- DT1 & DT2 = Delay time until a vehicle is available
- TT1 & TT2 = Vehicle transit time
- DL1 & DL2 = Delay time before vehicle is loaded
- TL1 & TL2 = Material load time
- DD1 & DD2 = Delay time prior to or during shipment
- TD1 & TD2 = Delivery time for shipment
- DW1 & DW2 = Waiting time before undelivered material is recorded
- DU1 & DU2 = Delay time prior to unloading material at user location
- TU1 & TU2 = Material unload time
  - PI = Probability that material is available at LSP
  - PS = Probability aht substitute is available at LSP, given material is not at LSP
- PD1 & PD2 = Probability that material is delivered, given that it was shipped
  - PC = Probability that material received is correct and undamaged material

Note that for the remaining parameters, the suffix 1 refers to the delivery from an LSP, and 2 refers to the delivery from an NLSP.

The process begins with the submittal of a material requisition that arrives at an inventory control point (ICP) after a time delay of TR time units. A search is then conducted to locate the material in inventory. However, it is assumed that the search, which takes an amount of time TF1, is not initiated until after some delay time DF1. The delay time would generally be due to a backlog of orders to process. If the material is in inventory at the LSP (with probability PI), the material is prepared for shipment, which takes TP1 time units, after another delay time of DP1. At that point transportation is scheduled, a process requiring TS1 time units. After some delay time DT1, the transportation vehicle is available and proceeds to the pickup point in the time TT1. Prior to loading, another delay time of DL1 may occur, after which the material is loaded in the time TL1. The transportation of the material to the user requires a minimum time of TD1, but a delay enroute of DD1 may also occur. Since the material being shipped may get lost enroute or be delivered to the wrong destination, there is a probability PD1 that the material will be delivered. At this time the material is unloaded in a time of TUl after some delay DUl. Since the material may or may not be useful as delivered, there is a probability PC that it is correct and undamaged, and the supply cycle is ended.

With probability 1-PC the material is incorrect or damaged and the supply cycle must be repeated with the submittal of another requisition.

If the material is not delivered (with probability 1-PD1), a certain waiting time passes before the material is reordered. This time is designated DW1.

If the material is not available at the LSP (with a probability 1-PI), then there is a possibility that a suitable alternate or substitute material can be provided from the LSP. The determination of this requires TF2 time units, and the probability of this event is PS.

With a probability of 1-PS, no substitute will be available at the LSP, and the material will have to be ordered from an NLSP. In this event, the determination of the location of the NLSP from which the material must be ordered requires TF3 time units. The remaining process

is assumed to be identical with the shipment of the material from an LSP except that the times required for the various steps may be different. These NLSP times are designated with the same notation as used for the LSP case, except that the final character in the notation is a 2 rather than a 1. Thus, for example, the material preparation time and the delay time are designed TP2, and DP2, respectively.

The process as described above can continue through the various steps a number of times, depending on the various branching probabilities. However, the probability of more than two iterations will be low, and for the purpose of the numerical example to be presented in the next section, we have assumed that after the second pass through the process, the correct material will be delivered either from the LSP or the NLSP.

The above restrictions permit us to calculate the expected response time by considering only nine cases. These cases are identified in Table IV-7. For each case, the response time equation and its corresponding probability of occurrence are indicated. At the end of Table IV-7, the system-response-time equation is given.

## C. The Cost Model

The cost MOEs are related to capital investment costs, C<sub>1</sub>, and operations and maintenance, Navy costs, C<sub>OMN</sub>. These costs are in turn broken down into the categories indicated in Table IV-8. Cost factors in capital investment are: R&D, facilities, working capital changes, and value of existing assets (employed or replaced). Cost factors in O&MN are personnel, maintenance and repair, materials, supplies, handling, etc., and overhead. The cost model considers only changes in these cost factors that would be expected in the future if a particular ED project is funded. The applicability of each type of cost factor to each ED project considered in this example is indicated by the entries in Table IV-8. Savings, when they occur, are indicated in parentheses. Note that for the value of existing assets (employed or replaced), only the values can be counted that result from the use or release of existing assets that can be gainfully employed in some other activity.

#### Table IV-7

#### RESPONSIVENESS MODEL EQUATIONS

### Case 1. Normal Delivery from LSP

 $T_{N} = TR + DF1 + TF1 + DP1 + TP1 + TS1 + DT1 + TT1 + DL1 + TL1 + DD1 + TD1 + DU1 + TU1$ 

 $P_N = (PI)(PD1)(PC)$ 

### Case 2. Incorrect or Damaged Delivery from LSP

$$T_D = 2 T_N$$

 $P_{D} = (PI)(PD1)(1 - PC)$ 

(We assume that if Case 2 occurs, it is followed by a Case 1 delivery with  $\operatorname{certainty}_{\:\raisebox{1pt}{\text{\circle*{1.5}}}}$ 

### Case 3. Material/Part Lost or Diverted in Shipment from LSP

 $T_{L} = T_{N} + TR + DF1 + TF1 + DP1 + TP1 + TS1 + DT1 + TT1 + DL1 + TL1 + DD1 + TD1 + DW1$ 

 $P_{\tau} = (PI)(1 - PD1)$ 

(We assume that if Case 3 occurs, it is followed by Case 1 with certainty.)

### Case 4. Normal Substitute Delivery from LSP

$$T_{NS} = T_{N} + TF2$$

 $P_{NS} = (1 - PI)(PS)(PD1)(PC)$ 

## Case 5. Incorrect or Damaged Substitute Delivery from LSP

$$T_{DS} = T_D + 2(TF2)$$

 $P_{DS} = (1 - PI)(PS)(PD1)(1 - PC)$ 

(We assume that if Case 5 occurs, it is followed by Case 4 with certainty.)

## Case 6. Substitute Lost or Diverted in Shipment from LSP

$$T_{LS} = T_L + 2(TF2)$$

$$P_{I,g} = (1 - PI)(PS)(1 - PD1)$$

(We assume that if Case 6 occurs, it is followed by Case 4 with certainty.)

### Case 7. Normal Delivery from NLSP

 $T_{NN} = TR + DF1 + TF1 + TF2 + TF3 + DP2 + TP2 + TS2 + DT2 + TT2 + DL2 + TL2 + DD2 + TD2 + DU2 + TU2$ 

 $P_{NN} = (1 - PI)(1 - PS)(PD2)(PC)$ 

### Table IV-7 (Concluded)

## Case 8. Incorrect or Damaged Delivery from NLSP

$$T_{DN} = 2 T_{NN}$$

$$P_{DN} = (1 - PI)(1 - PS)(PD2)(1 - PC)$$

(We assume that if Case 8 occurs, it is followed by Case 7 with certainty.)

# Case 9. Material/Part Lost or Diverted in Shipment from NLSP

$$T_{LN} = T_{NN} + TR + DF1 + TF1 + TF2 + TF3 + DP2 + TP2 + TS2 + DT2 + TT2 + DL2 + TL2 + DD2 + TD2 + DW2$$

$$P_{LN} = (1 - PI)(1 - PS)(1 - PD2)$$

(We assume that if Case 9 occurs, it is followed by Case 7 with certainty.)

# System Response Time

$$T_{R} = P_{N}T_{N} + P_{D}T_{D} + P_{L}T_{L} + P_{NS}T_{NS} + P_{DS}T_{DS} + P_{LS}T_{LS} + P_{NN}T_{NN} + P_{DN}T_{DN} + P_{LN}T_{LN}$$

Table IV-8 COST FACTORS

010	ED funds and R&D on transport system	Automated loaders, control systems, special support equipment	Modification and installation	Personnel training			(Reduced manpower)	Equipment maintenance		
65	ED funds		Modification of stocking/positioning procedures	Personnel training and stock repositioning	(Reduced inventory)	(Reduced warehouse space at LSPs) and increased warehouse space at NLSPs	(Reduced manpower at LSPs) and increased manpower at NLSPs		Increased handling and transportation costs	(Reduced overhead due to less warehouse space at LSP)
22	ED funds	Vehicles and sorting machines	Modifications of parcel handling procedures	Personnel training			Increased manpower	Vehicle maintenance and operation	(Reduced losses) and increased transportation costs) and elimination of postage)	Increased overhead due to more warehouse space at NLSPs
90	ED funds and development of manuals	Acquisition of manuals		Personnel training	(Reduced inventory)	(Some MHS equipment may be released for other use)			(Reduced handling and breakage)	
53	ED funds	Computers and software	Installation	Personnel training				Computer maintenance		
64	ED funds	Containers, loading equipment		Personnel training				Container repair	(Reduced handling losses and breakage)	
£3	ED funds				(Reduced inventory)	(fewer facilities)	(Reduced manpower)			(Reduced overhead due to closing facilities)
C1	ED funds	Computers and software	Installation	Personnel training			(Elimination of overtime)*	Computer maintenance		
Project Number Costs	Capital Investment R&D	Facilities Property and equipment	One-time services	Start-up costs	Working capital	Value of existing assetsemployed or replaced	OGMN Personnel (salary and benefits)	Maintenance and repair	Materials, supplies, handling, etc.	Overhead

\*Parentheses indicate savings.

The expected change in operations and maintenance, Navy cost  $(\Delta C_{OMN})$  , is computed on an annual basis and is given by

$$\Delta C_{OMN} = \Delta C_{PERS} + \Delta C_{M\&R} + \Delta C_{MSH} + \Delta C_{OH}$$
 (IV-1)

where

 $\Delta C_{PERS}$  = Expected annual cost change attributable to changes in personnel requirements

 $\Delta C_{M\&R}$  = Expected annual cost change attributable to changes in maintenance and repair requirements

ΔC<sub>MSH</sub> = Expected annual cost change attributable to changes in materials, supplies, handling, etc. requirements

 $\Delta C_{OH}$  = Expected annual cost change attributable to changes in overhead requirements.

Investment costs, for comparison of alternative systems, are usually normalized to an equivalent uniform annual investment cost over the expected economic lifetime of a system, since alternative systems will in general have unequal expected economic lifetimes. Thus, for this study, the investment cost MOE is the expected change in the equivalent uniform annual investment cost ( $\Delta C_T$ ) and is given by

$$\Delta C_{T} = (\Delta C_{R&D} + \Delta C_{FAC} + \Delta C_{WCC} + \Delta C_{A})/EL_{S}$$
 (IV-2)

where

 $\Delta C_{R\&D}$  = Expected research and development costs that would be incurred if a project is eventually funded

 $\Delta C_{FAC}$  = Expected investment cost changes attributable to changes in facilities requirements

 $\Delta C_{WCC}$  = Expected investment cost changes attributable to changes in working capital requirements

CA = Expected investment cost changes attributable to changes in existing assets requirements.

Expected system economic lifetime, as used here, refers to the expected length of time a system will operate without requiring major investment costs--e.g., replacement of computers that become obsolete or maintenance prone after so many years.

The cost MOEs ( $\Delta C_{O\!M\!N}$  and  $\Delta C_{I}$ ) indicated above apply to a single project. The ultimate MOEs used in applying the resource allocation methodology are the sums of each of these over all the projects included in a given ED alternative program.

#### V A SAMPLE RESOURCE ALLOCATION PROBLEM

To provide an example of how the resource allocation method would be applied, we went through the process of constructing a numerical example employing the responsiveness and cost MOEs applied to the supply system ED projects. For this example we first defined three different budget levels, and we then considered several alternative programs for allocating the funds within each budget.

## A. Budgets and Program Alternatives

The numerical example is based on an evaluation of the effects of the various add-on and new start projects, funded to the levels indicated in Table IV-2. Three incremental budget cases were considered. These consisted of the 25%, 50%, and 75% increment budgets. For each of these budget levels, the add-on and new start projects can be combined in various combinations to yield various alternatives called the 25%, 50%, and 75% Increment Programs. These are shown in Table IV-3.

For convenience, the add-on or new start projects comprising each alternative are shown in Table V-1. In Table V-1, add-ons are identified with a plus sign after the project designation.

From Table V-1 we see that there are four alternative 25% Increment Programs, five alternative 50% Increment Programs, and four alternative 75% Increment Programs. Since each of these alternatives is made up of various combinations of add-on projects, more combinations are available on the middle level of funding. It is important to note that there are many ways to form these alternative programs. In addition to combining them as we have, there is also the possibility of trading off funding between or among projects in various ways (this may be feasible only for certain projects).

Table V-1
SUPPLY SYSTEM ALTERNATIVE ED PROGRAMS

25% Increment Pr	ograms				
Alternative	I	II	III	IV	
Project	C4+	C4+	С9	C10	
Project	C5+	C7+			
50% Increment Pr	rograms				
Alternative	I	II	III	IV	V
Project	C4+	C4+	C4+	C4+	С9
Project	C5+	C5+	C7+	C7+	C10
Project	С9	C10	С9	C10	
75% Increment Pi	cograms				
Alternative	I	II	III	IV	
Project	C4+	C4+	C5+	C7+	
Project	C5+	C5+	С9	C9	
Project	C7+	C7+	C10	C10	
Project	С9	C10			

By combining the effects of individual project add-ons or new starts according to the combinations in Table IV-3, we were able to derive estimates of the effects of each alternative program on the seven MOEs presented in Section IV.

# B. Responsiveness MOE Values

We built up the numerical example by first estimating the present levels of all the responsiveness MOEs. These numerical values as well as all others developed are simply educated guesses, since it was not the purpose of this initial research to conduct the more detailed analysis required to establish these numbers with more confidence. In any real application of this method, the appropriate supporting analysis must be carried out.

Table V-2 shows our estimates of the present levels of the response parameters identified in the responsiveness model (Section IV-A). The response time units are in minutes of working day time. In Table V-2, several response time parameters are grouped on one line, and each line is numbered with a double-digit number. The reason for these groupings was to fit the memory constraints of the Texas Instruments SR-52 calculator. The responsiveness model equations were programmed on the SR-52 to facilitate the calculation of the expected response time in each scenario. The SR-52 advertises 20 addressable memory locations, but actually has two additional memory locations. The first 20 are addressed as 00 through 19, and the other two are 98 and 99. Thus, in Table V-2, each line corresponds to a particular SR-52 memory location with the indicated double-digit address. This program is listed in Appendix B.

Scenario Al is the case of a small requisition from either an LSP or an NLSP. This type of scenario involves all the individual response times in the responsiveness model. Thus, in Table V-2, all response time and probability parameters have been assigned an estimated value.

Scenario A2 is the repair-at-sea requisition from an LSP or NLSP. The LSP in this case is simply the ship's own organic repair supplies, and the NLSP is another ship within the same task force or group. Thus, A2 is similar to A1 except for the inherent close proximity of the user and the supplier, and the criticality of the repair need. In particular, we have set the probability of non-delivery from the LSP, given the availability of the required part (1-PD1) equal to zero. Thus, the value of DW1, the waiting time before reordering, is irrelevant, and we have set it equal to zero. In addition, there are a number of delay time parameters that have been set equal to zero to account for the immediacy of the need for repair when at sea.

Scenario B is the heavy equipment requisition from the NLSP. In this scenario all the response time and probability parameters relating

Table V-2
RESPONSE PARAMETER VALUES--PRESENT LEVEL

Number	Responsiveness Parameters			Scenario		
	(time units = min)	A1	A2	В	υ	D
00	TR + DF1 + TF1	60 + 10 + 5	60 + 10 + 5	60 + 10 + NA*	60 + NA + NA	480 + NA + NA
10	TF2	15	15	NA	NA	NA
02	TF3	15	09	09	180	1440
03	DPI + TPI	120 + 10	0+0	NA	NA	NA
70	DP2 + TP2	120 + 10	30 + 10	120 + 480	0 + 240	0 + 2400
90	TSI	5	0	NA	NA	NA
90	TS2	5	5	5	5	09
07	DT1 + TT1	120 + 20	15 + 2	NA	NA	NA
80	DT2 + TT2	480 + 20	15 + 0	480 + 20	480 + 20	7200 + NA
60	DLI + TLI	15 + 1	15 + 1	NA	NA	NA
10	DL2 + TL2	15 + 1	15 + 1	30 + 180	15 + 360	480 + 2400
11	DD1 + TD1	0 + 15	0 + 15	NA	NA	NA
12	DD2 + TD2	60 + 180	0 + 15	009 + 09	009 + 09	1440 + 4800
13	DW1	480	0	NA	NA	NA
14	DW2	1440	240	2880	NA	NA
15	DU1 + TU1	15 + 15	0 + 5	NA	NA	NA
16	DU2 + TU2	15 + 15	15 + 15	480 + 240	480 + 240	120 + 480
17	PI	0.85	0.85	0	0	0
18	PS	09.0	0.70	0	0	0
19	PD1	0.80	1.0	NA	NA	NA
86	PD2	0.80	0.95	0.95	1.0	1.0
66	PC	09.0	0.95	0.95	1.0	1.0
Expected Re	Expected Response Time					
Minutes		815.0	135.9	3208.4	2740.0	21300.0
Working days	days	1.70	0.28	6.68	5.71	44.38

\*NA = Not Applicable.

to delivery from the LSP do not apply, as indicated by the dash in the appropriate box in Table V-2. This is equivalent to a zero value of these parameters in the expected response time computation. In addition, PI and PS, the probabilities of availability at the LSP of the required material or substitute, respectively, must be zero according to the scenario assumption.

Scenario C is the high-volume supply requisition from the NLSP. Again, in this scenario, the response times and probabilities relating to delivery from the LSP do not apply. In addition, this type of supply function is handled in a "push" mode where deliveries are scheduled at preplanned intervals. Thus, the user cannot directly relate to the responsiveness of the system. However, from the supplier's viewpoint, the responsivness of the system is quite pertinent. The more sluggish the system, the more resources he must employ to meet the required supply schedules. This can be partly measured in terms of the costs involved in operating the supply system. In addition, the responsiveness of the system will certainly affect the throughput, or amount of supply material that can be processed through the system over a given period of time with a given set of available resources. Thus, in retrospect, a more suitable unit of measuring the responsiveness of a "push" type of supply system should be developed. However, for the purpose of this example, we will consider only the expected response time defined from the viewpoint of the supplier rather than the user. This must be distinguished from the response times in the previous three scenarios, which are defined from the user's point of view.

In a "push" supply system several other response parameters do not apply. One such parameter is DW2, the waiting delay time before reordering supplies that have not been delivered. This situation is handled in the responsiveness model by setting PD2 = 1.0 so that reordering does not occur. Similarly, PC must also be set equal to unity to prevent the reordering cycle from occurring. In the initiation of the "push" supply cycle, only two processing times are assumed to be relevant. First, there is a certain amount of paper-processing time to initiate the supply cycle, and a certain amount of time to locate the required material.

For Scenario C, these times are designated TR, and TF3, respectively, and DF1, TF1, and TF2 are assumed not to apply.

The last scenario, Scenario D, is the underway replenishment (UNREP) case. Here again the supplies are delivered from an NLSP, the shore supply facilities. Thus, the response time and probability parameters for the LSP case do not apply, and PI and PS are zero. As in Scenario C, this type of resupply is handled in a "push" mode, and response time must be interpreted from the supplier's point of view.

To obtain estimates of the effects of the various budget levels and the corresponding ED programs, we first developed a table showing our estimates of which response times in which scenarios are affected by each of the supply system projects. This information is shown in Table V-3. Using this table, we were able to develop the subsequent tables, which show our estimates of the quantitative effects of the various programs.

In Table V-3, each ED project is listed on the left-hand side, and each column to the right corresponds to a particular scenario. Blanks indicate that the corresponding project has no effect on any of the response parameters in the corresponding scenario. Where effects do occur, the response parameters are identified, and the direction of the effect is indicated by a plus or minus to indicate an increase or decrease. Negative effect on response times and positive effect on probabilities are preferred. Note that in some cases we get the opposite effect. For example, under Scenario C for project C3, the Material Distribution Study, we can anticipate that one effect may be to consolidate supply functions by closing down some supply facilities. In such a case, we can expect that DD2 and TD2, the response times associated with transporting material from an NLSP, will be increased due to increased distances involved. This effect should be more than compensated for by improvements in other response parameters, or costs, or both.

After establishing the present MOE levels, we estimated the effects of the present funding of the ED projects. This was called the Base Program case. This case consists of the present funding budget shown

Table V-3

ADD-ON PROJECTS EFFECTS ON RESPONSE PARAMETERS

ED			S	cenario			
Projects	A1		A	.2	В	С	D
C1	-TS1 -TS2 -DT1	-TT1 -DT2 -TT2					
С3	-P1 +P5					-TR -TF3 +DD2 +TD2	-TR -TF3
C4						+TP2 -TU2 -TL2	+TP2 -TU2 -TL2
C5			-DF1 -TF1 -TF2 -TF3	-DT1 -TT1 +PC			-DU2
C6	-DF1 -TF1 -TF2 -TF3 -DP1 -TP1	-DP2 -TP2 -DL1 -DL2 +PC			-DF1 -DP2 -TP2 -DL2 +PC	-TF3 -TP2 -DL2	-TF3 -TP2 -DL2
C7	-DD2 -TD2 -DT2 -TT2	-TS2 +PD1 +PD2 +PC					
С9	+DI +TI -P:	)2		DD2 CD2 PI			
C10						-TP2 -DL2 -TL2	-TP2 -DL2 -TL2

in Table IV-2. These effects are shown in Table V-4. In Table V-4, only the changes in the response parameters affected are shown. Table V-5 shows the new response parameter values for the Base Program case obtained after applying the changed indicated in Table V-4 to the values in Table V-2. Also shown at the bottom are the new expected response times, and the changes in response time from the present level.

The next step in the process consisted of estimating the changes in response parameters caused by each of the various add-on projects. These may be due to additional funding of previously funded projects or the introduction of new projects (C9 and C10). These changes are shown in Table V-6, and the changes are relative to the Base Program response parameter values. In certain cases, more than one add-on project may affect the same response parameters. When this occurs, and when we consider the alternative of funding both projects, a combined effect must be estimated. These combined effects are also shown in Table V-6 and are not necessarily the sum of the individual effects.

We note in Table V-6 that no changes occur in Scenario B due to any of the add-on projects. This means that the expected response time in Scenario B does not enter into the choice from among the set of alternatives. Scenario B will therefore be dropped from the sample problem. However, if this were actually the case, the decision maker might want to ask his staff to construct a new alternative program that would affect the response time in Scenario B. This would require the introduction of one or more new projects.

Using the information in Table V-6 to modify the Base Program values in Table V-5 according to the various alternative programs defined in Table V-1, we can complete our estimates of all response value parameters. The results of this process are summarized in Table V-7. Here we show the expected response times for each of the scenarios and for each alternative program. As a reference, we have also shown the results for the present level, base Program, and 100% Increment Program. The 100% Increment Program case represents only one alternative way to allocate the entire add-on budget. Thus, it will not enter into any tradeoff assessments employing the proposed method.

Table V-4

RESPONSE PARAMETER CHANGES -- BASE PROGRAM

А	Responsiveness Parameters			Scenario		
Number	(time units = min)	Al	A2	В	C	Q
00	TR + DF1 + TF1	0, -5, -2	0, -5, -2	0, -5, 0	-15, 0, 0	0 '0 '09-
01	TF2	-5	89	0	0	0
02	TF3	-5	-25	0	-90	-240
63	DP1 + TP1	-40, -2	0	0	0	0
8	DP2 + TP2	-40, -2	0	-40, -120	0, +30	0, +240
05	TS1	-2	0	0	0	0
90	TS2	7	0	0	0	0
07	DT1 + TT1	-40, -10	-4, -1	0,0	0,0	0
80	DT2 + TT2	-120, -5	0,0	0,0	0,0	0,0
60		-5, 0	0,0	0,0	0,0	0
10		-5, 0	0,0	-10, 0	-10, -60	-60, -300
11		0,0	0,0	0,0	0,0	0
12		-5, -20	0,0	0,0	+30, +120	0,0
13	DW1	0	0	0	0	0
14	DW2	0	0	0	0	0
15		0,0	0 0	0 0	0 0	0
16	DU2 + TU2	0,0	0 0	0,0	0, -30	-30, -30
17	PI	-0.05	0	0	0	0
18	PS	+0.05	0	0	0	0
19	PD1	+0.06	0	0	0	0
86	PD2	+0.06	0	0	0	0
66	PC	+0.08	+0.02	+0.03	0	0

Table V-5
RESPONSE PARAMETER VALUES--BASE PROGRAM

ΩI	Responsiveness Parameters			Scenario		
Number	(time units = min)	A1	A2	В	C	D
00	TR + DL1 + TF1	60 + 5 + 3	60 + 5 + 3	0 + 5 + 09	0 + 0 + 57	420 + 0 + 0
0.01	TF2	10	7	0	0	0
02	TF3	10	35	09	06	1200
03	DP1 + TP1	8 + 08	0 + 0	0	0	0
8	DP2 + TP2	8 + 08	30 + 10	80 + 360	0 + 270	0 + 2640
05	TS1	3	0	0	0	0
90	TS2	4	2	2	0	09
07	DT1 + TT1	80 + 10	11 + 1	0	0	0
80	DT2 + TT2	360 + 15	15 + 0	480 + 20	480 + 20	7200 + 0
60	DL1 + TL1	10 + 1	15 + 1	0	0	0
10	DL2 + TL2	10 + 1	15 + 1	20 + 180	5 + 300	420 + 2100
11	DD1 + TD1	0 + 15	0 + 15	0	0	0
12	DD2 + TD2	55 + 160	0 + 15	009 + 09	90 + 720	1440 + 4800
13	DW1	480	0	0	0	0
14	DW2	1440	240	2880	0	0
15	DUI + TUI	15 + 15	0 + 15	0	0	0
16	DU2 + TU2	15 + 15	15 + 15	480 + 240	480 + 210	90 + 450
17	PI	08.0	0.85	0	0	0
18	PS	0.65	0.7	0	0	0
19	PD1	98.0	1.0	0	0	0
86	PD2	98.0	0.95	0.95	1.0	1.0
66	PC	0.68	0.97	96.0	1.0	1.0
Expected	Expected Response Time					
Minutes		579.5	118.5	2940.9	2805.0	20820.0
Working days	days	1.21	0.25	6.13	5.84	43,38

Table V-6
RESPONSE PARAMETER CHANGES--ADD-ON PROJECTS

ID	Responsiveness Parameters			Scenario		
Number	(time units = min)	A1	A2	В	С	D
00	TR + DF1 + TF1		C5* 0, -3, -2			
01	TF2		C5 -4			
02	TF3		C5 -15			
04	DP2 + TP2				C4 0, +30 C10 0, -60 C4 + C10 0, -30	C4 0, +240 C10 0, -60 C4 + C10 0, -360
06	TS2	C7 -1				
07	DT1 + TT1		C5 -4, 0			
08	DT2 + TT2	C7 -30, -5				
10	DL2 + TL2				C4 -5, -60 C10 -5, -60 C4 + C10 -5, -90	C4 -60, -300 C10 -60, -700 C4 + C10 -90, -900
12	DD2 + TD2	C7 -5, -10 C9 0, +40 C7 + C9 -5, +30	C9 0, +10			
16	DU2 + TU2				C4 0, -30	C4 0, -30 C5 -30, 0 C4 + C5 -30, -30
17	PI	C9 -0.05	C9 -0.10			
19	PDI	C7 +0.04				
98	PD2	C7 +0.04				
99	PC	C7 +0.07	C5 +0.02			

 $<sup>^{\</sup>mbox{\tiny $^{\circ}$}}\text{C4, C5, C7, C9, and Cl0 are project numbers.}$ 

Table V-7
EXPECTED RESPONSE TIMES

		Scen	ario	
	Al (working hours)	A2 (working hours)	C (working days)	D (working days)
Base Programs	9.7	1.98	5.84	43.4
25% Increment Programs				
Alternative I	9.7	1.76	5.71	43.0
Alternative II	8.7	1.98	5.71	43.1
Alternative III	10.1	2.09	5.84	43.4
Alternative IV	9.7	1.98	5.58	40.5
50% Increment Programs				
Alternative I	10.1	1.86	5.71	43.0
Alternative II	9.7	1.76	5.52	40.4
Alternative III	9.1	2.09	5.71	43.1
Alternative IV	8.7	1.98	5.52	40.5
Alternative V	10.1	2.09	5.58	40.5
75 Increment Programs				
Alternative I	9.2	1.86	5.71	43.0
Alternative II	8.7	1.76	5.52	40.4
Alternative III	10.1	1.86	5.58	40.5
Alternative IV	9.2	2.09	5.58	40.5
100% Increment Programs	9.1	1.86	5.52	40.4

Note that in Table V-7 we have modified the response time units to working hours for Scenarios Al and A2, and to working days for Scenarios C and D. (A working day is assumed to consist of 480 working minutes.) This was done because of the large spread of response times when dealing strictly in working minutes.

A review of the values shown in Table V-7 shows that although funding of the 100% Increment Program improves the responsiveness in all scenarios, improvements are not obtained in all cases for all intermediate alternatives. For example, comparing Alternative III for the 25% Increment Program to the Base Program, we see that the response time in Scenarios Al and A2 is expected to increase with additional funding. This is primarily because this particular alternative involves adding on project C9, which is the Multi-Echelon Repairables Project. This will generally require transporting the requisitioned material over a larger distance than previously, and thus increase the response time. The benefit of C9 will be primarily in cost savings, and these savings may well override the degraded response time.

We also see from Table V-7 that certain alternatives dominate others in response time for all scenarios. At the 25% Increment Program, Alternative I dominates III. At the 50% Increment Program, Alternative II dominates both I and V. Finally, at the 75% Increment Program, Alternative II dominates I, III, and IV. However, since cost savings have not yet been considered, we cannot as yet reach any conclusions.

The values in Table V-7 for the various alternative programs are the responsiveness MOEs required as input to the resource allocation method. As discussed earlier, the expected response time in each scenario corresponds to a unique MOE. Thus, Table V-7 provides four of the MOEs required for ranking the supply system programs.

Two other MOEs are required before the RA method can be applied. These are the capital investment, and OM&N costs. Estimates for these costs were generated next.

### C. Cost MOE Values

The cost model described in Section IV is designed to generate values for incremental changes in equivalent uniform annual investment costs and annual O&MN costs. For the example of this section, the alternative ED programs of concern are defined in terms of incremental ED budget increases over and above that which is already committed -- that is, the base budget as previously defined by the present funding in Table IV-2. Thus, values for the cost MOEs  $\Delta C_T$  and  $\Delta C_{OMN}$  need be estimated only for those ED projects that are specified for possible add-on funding (C4, C5, C7, C9, and C10), and these values should reflect changes attributable only to effects of the add-on funding increments. For ED projects C9 and C10, these are both totally included as add-ons and thus can be handled in a straightforward manner. For ED projects C4, C5, and C7, the add-on funding represents increases in funds already committed. For these latter cases, the changes attributable to the add-on funding can best be estimated by estimating expected MOE values attributable to, first, the present funding, and second, the total funding (present plus add-on), and then subtracing the former from the latter.

The cost MOE values are derived on the basis of "off the cuff" assumptions as to future ramifications for the Naval supply system that might result from implementation of the results of the ED projects. These assumptions are not based on any analysis and simply are reflections of what might be. Table V-8 presents a number of these assumptions that indicate some ball-park estimates of planning factors that are used as a basis for applying cost factor estimates to obtain the values for the cost MOEs.

The cost factor assumptions, peculiar to each ED project, used to generate the cost MOE estimates are delineated in Table V-9. As an example only, the following assumptions were made to segregate the future effects of the two-stage funding projects. For Project C4, it was assumed that the present level of ED funding would lead to a medium-sized container system while the inclusion of add-on ED funding would lead to a large-sized container system. For Project C5, it was assumed that the system resulting from the present level of ED funding would be computerized

Table V-8
PLANNING FACTOR ASSUMPTIONS

Planning Factor Area	Planning Factor Assumptions
Supply points	25 NLSPs (supply centers, supply depots, air stations, and shipyards)  131 LSPs (all supply activities including NLSPs)
Naval ships affected by ED projects	6 Logistic ships 213 Combatant ships (excludes submarines)
Inventory	\$1.7 B total affected inventory (excludes low-quantity items and afloat required spares) \$170 M annual loss of inventory \$17 M annual breakage of inventory
Small parcels	10 M parcels handled per year 1000 miles average delivery distance 5 1b average weight per parcel

Table V-9
COST FACTOR ASSUMPTIONS

ED Project	Cost Factor Assumptions
C4 Present funding only (\$100K)	INVESTMENT COSTS  No additional R&D costs above ED costs  10 medium containers per NLSP (\$3K per container)  1 medium container loading system per NLSP (\$10K per system)  Training of 6 personnel per NLSP (\$2.5K per person)  15 year estimated system economic lifetime  O&MN COSTS  System maintenance & repair costs (11% of equipment investment cost per year)  20% reduction in losses and breakage
Total Funding Present + Add-on (\$150K)	INVESTMENT COSTS  No additional R&D costs above ED costs  10 large containers per NLSP (\$6K per container)  1 large container loading system per NLSP (\$30K per system)  Training of 6 personnel per NLSP (\$2.5K per person)  15 year estimated system economic lifetime  O&MN COSTS  Maintenance & repair costs (11% of equipment investment cost per year)  40% reduction in losses and breakage
C5 Present funding only (\$210K)	INVESTMENT COSTS  No additional R&D costs above ED costs  1 large computer system per logistics ship (\$65K per computer) Installation of computer systems (\$5K per system) Training of 2 personnel per logistics ship (\$1K per person) 10 year estimated system economic lifetime O&MN COSTS Computer system maintenance & repair costs (10% of total equipment investment cost per year)
Total Funding Present + Add-on (\$410K)	INVESTMENT COSTS  No additional R&D costs above ED costs  1 large computer system per logistics ship (\$65K per computer)  1 small computer system per combatant ship (\$30K per computer)  Installation of computer systems (\$5K per large computer & \$1K per small computer  Training of 2 personnel per logistics & combatant ship (\$1K per person)  O&MN COSTS  10 year estimated system economic lifetime  Computer system maintenance & repair costs (10% of total equipment investment  cost per year)
C7 Present funding only (\$80K)	INVESTMENT COSTS  No additional R&D costs above ED costs  Parcel handling procedural changes (\$2K per LSP)  Training of 10 personnel per LSP (\$1K per person)  15 year estimated system economic lifetime  O&MN COSTS  4 additional personnel per LSP (\$30K per person per year)  10% reduction in inventory losses per year
Total Funding Present + Add-on (\$280K)	INVESTMENT COSTS  No additional R&D costs above ED costs  2 delivery vehicles per LSP (\$12K per vehicle)  Parcel handling procedural changes (\$2K per LSP)  Training of 10 personnel per LSP (\$2.5K per person)  6 sorters per LSP (\$5K per sorter)  15 year estimated system economic lifetime  O&MN COSTS  4 additional personnel per LSP (\$30K per person per year)  Sorter & vehicle maintenance & repair costs (5% of equipment investment cost/year Increased transportation costs (\$.00016 per pound per mile)  Reduced postage rates (\$1.50 per parcel)  50% reduction in inventory losses per year

## Table V-9 (Concluded)

## COST FACTOR ASSUMPTIONS

ED Project	Cost Factor Assumptions
С9	INVESTMENT COSTS
Total Funding (\$300K)	No additional R&D costs above ED costs Procedural changes (\$4K per LSP)
	Training of 10 personnel per LSP (\$1K per person)
	Resposition of stockage (\$200K)
	10% reduction in inventory
	Decreased facility requirement at each non-NLSP LSP (\$50K per LSP)
	Increased facility requirements at each NLSP (\$100K per NLSP)
	15 year estimated system lifetime OSMN COSTS
	Reduction of 3 personnel at each non-NLSP LSP (\$30K per person)
	Increase of 2 personnel at each NLSP (\$30K per person)
	Increased handling & transportation costs (10% of inventory savings)
	Facility overhead (75% of facility requirements costs and/or savings)
C10	INVESTMENT COSTS
Total Funding	\$750K R&D costs in addition to ED costs
(\$300K)	1 system per NLSP (\$100K per system)
	System installation (\$30K per system)
	Training of 12 personnel per NLSP (\$5K per person)
	15 year estimated system economic lifetime
	OSMN COSTS
	Reduction of 10 personnel per NLSP (\$30K per person per year)  System maintenance & repair costs (15% of total equipment investment cost/year)

only aboard logistics ships, while the inclusion of the add-on funding would result in automation aboard combatant ships also. Finally, for Project C7, the present level of ED funding is assumed to lead to only parcel handling procedural changes at LSPs, while the add-on ED funding would presumably lead to a total parcel handling system within the Naval Supply system, which includes base-to-base transport of parcels and on-base delivery services.

Using the planning factor and cost factor assumptions of Tables V-8 and V-9, respectively, as inputs to the cost model (Section IV), the values of the cost MOEs associated with each add-on ED project were computed. The results are presented in Table V-10. These results then provided the basis for computing the cost MOE values for the alternative ED budget allocations by appropriately combining the project MOE values in accordance with the alternative allocations as defined in Table V-1. The cost MOE values for these alternative ED budget allocations are presented in Table V-11.

## D. Application of the RA Method

In the previous sections we derived numerical examples for three incremental budgets. Each budget level contained four to five alternative ways of allocating the funds to the various add-on ED projects. For each alternative, we estimated values for six MOEs, four of which were supply response times in different scenarios, and two were costs (or savings) resulting from successful completion of the projects. The costs were broken down into annualized capital investment costs, and annual operations and maintenance, Navy, costs. Table V-12 summarizes the estimated MOE values for each alternative. In Table V-12, MOEs are shown in response time increase and cost increase from the Base Program case. Thus, negative values indicate time reductions and dollar savings, and negative values are preferred to positive values.

The changes in response times are shown in minutes for Scenarios Al, A2, and C, and in hours for Scenario D in order to work with convenient magnitudes. Cost changes are shown in millions of dollars.

Table V-10
COST MOE ESTIMATES FOR ADD-ON PROJECTS

Add-On ED Project	Annual Expected Uniform Investment Cost Change	Annual Expected O&MN Cost Change ΔΟ&MN (thousands of dollars per Year)†
C4	\$ 87	(\$37,262)
C5	723	639
C7	616	(74,646)
C9	(11,364)	6,860
C10	387	6,875

 $<sup>^{\</sup>star}$ Parenthesis indicate decreases in annual costs.

For the 25% Increment Program we see that no alternative dominates another in preference for all MOEs.\* However, Alternative III dominates the others only in C<sub>I</sub>, the capital investment costs. In particular, when we compare Alternative II to III, the large savings in C<sub>OMN</sub> for II, and the reduced response times in three out of four scenarios, would probably argue for an a-priori preference of II over III. Similarly, one can argue that I may also be preferred to III. Preference between III and IV may be more debatable. Comparing I and II to IV may argue for a bias toward I or II due to the large cost savings achieved by both. Thus, a DM may indicate an a-priori preference for the pair I and II compared to all other possible pairs. These then appear to be likely candidates for Step 1 in the tradeoff assessment method.

<sup>&</sup>lt;sup>†</sup>Attributable to add-on funding only.

<sup>\*</sup>The preferences indicated in the remainder of this section are based on the six selected MOEs. Clearly, use of other MOEs such as life cycle cost could alter these preferences.

Table V-11

COST MOE\* ESTIMATES FOR ALTERNATIVE ED BUDGET ALLOCATIONS

	ΔC <sub>T</sub>	$\Delta^{\mathbf{C}}_{\mathbf{OMN}}$
	(thousands of dollars per year) <sup>†</sup>	(thousands of dollars per year) <sup>†</sup>
25% Increment Programs		
Alternative I	\$ 810	(\$36,623)
Alternative II	703	(111,908)
Alternative III	(11,364)	6,860
Alternative IV	387	(6,875)
50% Increment Programs		
Alternative I	(10,554)	(29,763)
Alternative II	1,197	(43,498)
Alternative III	(10,661)	(105,048)
Alternative IV	1,090	(118,783)
Alternative V	(10,977)	(15)
75% Increment Programs		
Alternative I	(9,938)	(104,409)
Alternative II	1,813	(118,144)
Alternative III	(10,254)	625
Alternative IV	(10,361)	(74,661)
100% Increment Programs	(9,551)	(111,284)

<sup>\*</sup>In these cases, investment and O&MN cost changes have been considered separately. A combined MOE such as life-cycle cost could also be considered.

<sup>†</sup>Parenthesis indicate decreases in annual costs.

Table V-12

EXPECTED MOE VALUES FOR THE SELECTED BUDGETS AND ALTERNATIVES

		Response Ti	Response Time Changes		Cost C	Cost Changes
	∆R <sub>A1</sub>	∆R <sub>A2</sub>	D <sub>M</sub> C	ARD	1 <sub>DV</sub>	∆C <sub>OMN</sub>
	(working minutes)	(working minutes)	(working minutes)	(working hours)	(millions of dollars)	(millions of dollars)
25% Increment Programs						
Alternative I*	0	-13.2	-7.8	-3.2	0.8	-36.6
Alternative II*	09-	0	-7.8	-2.4	0.7	-111.9
Alternative III	24	9.9	0	0	-11.4	6.9
Alternative IV	0	0	-15.6	-23.2	0.4	6.9-
50% Increment Programs						
Alternative I	24	-7.2	-7.8	-3.2	-10.6	-29.8
Alternative II*	0	-13.2	-19.2	-2.4	1.2	-43.5
Alternative III	-36	9.9	-7.8	-2.4	-10.7	-105.0
Alternative IV*	09-	0	-19.2	-23.2	1.1	-118.8
Alternative V	24	9.9	-15.6	-23.2	-11.0	-0.02
75% Increment Programs						
Alternative I	-30	-7.2	-7.8	-3.2	6.6-	-104.4
Alternative II*	09-	-13.2	-19.2	-2.4	1.8	-118.1
Alternative III	24	-7.2	-15.6	-23.2	-10.3	9.0
Alternative $\mathrm{IV}^{\!\star}$	-30	9.9	-15.6	-23.2	-10.4	-74.7

\* A-priori preferred alternatives.

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For the 50% Increment Program, no alternative dominates another in preference for all MOEs. Alternative IV dominates all other alternatives in four out of the six MOEs, and shows a large improvement in  $C_{OMN}$  &  $R_{D}$ . Although it dominates only two out of the remaining four alternatives in  $R_{\Delta\,2}$ , it neither increases nor decreases this response time. It is primarily in C<sub>T</sub> that IV does not perform well. However, the fact that savings in  $C_{\overline{OMN}}$  are about 10 times the best savings in  $C_{\overline{I}}$ , argues that Alternative IV would be ranked high. If we compare III to I, we get a significantly larger improvement in  $C_{\mbox{\scriptsize OMN}}$  for III, about equal performance in  $R_C$ ,  $R_D$ , and  $C_T$ , and significant improvement in  $R_{A1}$  for III. Alternative III does not perform as well as I in  $R_{\Delta 2}$ ; however, the magnitudes in response time improvements are not that large. We may then argue that III would probably be preferred to I. Comparing II to V, II performs better in all but  $C_T$ . However, in total dollar savings ( $C_T$  + Commu, II performs better than V. Thus, we might argue for preference of II over V. At this point, II, III, and IV appear to be the set of potentially higher-ranking alternatives. Finally, comparing II to III, we see that III performs significantly better in dollar savings, while II performs significantly better in  $R_{D^{\bullet}}$  The relative importance of improvement in R<sub>n</sub> versus dollar savings might be the determining factor between II and III. We assume the  $R_{
m D}$  is more important in this case and argue for II. Thus, Alternatives II and IV appear to be likely starting candidates for application of the method to the 50% Increment Program case.

Finally, for the 75% Increment Program, again no alternative dominates another in preference for all MOEs. Comparing II to all other alternatives shows that II dominates in all MOEs except  $C_{\rm I}$ . However, the large savings in  $C_{\rm OMN}$ , and the fact that the sum of  $C_{\rm I}$  and  $C_{\rm OMN}$  for II dominates all other alternatives, argues for the selection of II as the a-priori most preferred. Comparing III to IV, we see that IV dominates or equals III in all but  $R_{\rm A2}$ . The large savings in  $C_{\rm OMN}$  for IV when compared with the magnitude of improvement in  $R_{\rm A2}$  for III would argue for selection of IV over III. Comparing I to IV shows that IV dominates in four out of six MOEs, and has a significantly larger

improvement in  $R_D$  compared to I. The dominance of I over IV in  $C_{O\!M\!N}$  is not strong on a percentage improvement basis, although the magnitude of the difference in savings is substantial ( $\approx$ \$25M). Arguing as before that improving  $R_D$  may be relatively more important than improving  $C_{O\!M\!N}$ , leads to a tentative ranking of IV over I. Thus, we arrive at II and IV as the likely starting candidates for Step 1 in the method.

These tentative choices, for each budget, are marked in Table V-12 with an asterisk.

Proceeding to Step 2, for the 25% Increment Program case, we observe that Alternative I dominates II in two MOEs ( $R_{A2}$  and  $R_{D}$ ), while II dominates I in three MOEs ( $R_{A1}$ ,  $C_{I}$ , and  $C_{OMN}$ ). Both alternatives are equal in  $R_{C}$ . The MOEs will thus be reordered with  $R_{A2}$  and  $R_{D}$  in the first two positions. The relative order between  $R_{A2}$  and  $R_{D}$  can be selected on the basis of other criteria. For example, since  $R_{A1}$  and  $R_{A2}$  are for similar scenarios, we may perform the reordering so that these MOEs are consecutive, and will be involved in one of the tradeoff assessments to come. Thus, we reorder these MOEs in the following sequence:  $R_{D}$ ,  $R_{A2}$ ,  $R_{A1}$ ,  $C_{T}$ ,  $C_{OMN}$ , and  $R_{C}$ .

Step 3 consists of constructing the tradeoff assessment tableau, and this is shown for Alternatives I and II in Table V-13. A minimum of one tradeoff assessment will be required to determine preference between I and II. This can be seen by considering the hypothetical alternative I' and the DMs response to be inserted in the blank box for MOE  $R_{A2}$ . If  $\Delta R_{A2}$  is greater than 0, then II is indifferent to I' in  $R_D$  and  $R_C$ , but dominates in all remaining MOEs. Thus, II is preferred to I' and must also be preferred to I. If  $\Delta R_{A2}$  is exactly 0, then II will still dominate I' in 3 out of 6 MOEs and be indifferent in the other 3. Thus, again II is preferred to I.

On the other hand, if a number less than 0 is required for MOE  $R_{A2}$ , we cannot yet determine preference between the alternatives, and must proceed to at least one more tradeoff assessment. In any case after at most four tradeoff assessments we will be able to determine the DMs preference by comparing the value of MOE  $C_{O\!M\!N}$  for I''' to that for II.

Table V-13

25% INCREMENT PROGRAM TABLEAU

$II \sim I$	$\begin{array}{c c} \Delta C_{OMN} & \Delta R_C \\ \text{(millions} & \text{(working)} \\ \text{of dollars)} & \text{minutes)} \end{array}$	-36.6 -7.8	-36.6 -7.8	-36.6 -7.8	-36.6 -7.8	-7.8	-111.9
1 < 11	$\Delta C_{ m I}$ (millions of dollars)	8.0	8.0	8.0		0.7	7.0
	$\begin{array}{c} \Delta R_{A1} \\ \text{(working} \\ \text{minutes)} \end{array}$	0	0		09-	09-	09-
11	ARA2 (working minutes)	-13.2		0	0	0	0
11 < 1	$\begin{array}{c} \Delta R_{D} \\ \text{(working hours)} \end{array}$	-3.2	-2.4	-2.4	-2.4	-2.4	-2.4
	Alternatives	(Real) I	(Hypothetical) I'	(Hypothetical) I''	(Hypothetical) I'''	(Hypothetical) I''''	(Real) II

MOE  $R_{\mbox{\scriptsize C}}$  does not enter into the preference assessment between I and II, since it is equal for both.

Tables V-14 and V-15 show the initial tableaus for the 50% Increment Program and 75% Increment Program, respectively. For the 50% Increment Program,  $R_{\mbox{\scriptsize C}}$  again does not enter into preference assessment between II and IV. The minimum and maximum number of tradeoff assessments required are one and four, respectively.

For the 75% Increment Program, we see that all MOEs must be considered in the preference assessment. Since Alternative II dominates IV in all but one MOE, at least one tradeoff assessment and at most five tradeoff assessments will be required.

The next step in the procedure, Step 4, consists of asking the DM to assess his tradeoffs between pairs of MOE for two distinct alternatives. For example, the first pair of MOEs to consider for the 25% Increment Program case (Table V-13) is response time in the UNREP scenario,  $\mathbf{R}_{\mathbf{D}}$ , and response time in the repair-at-sea scenario,  $\mathbf{R}_{\mathbf{A}2}$ . We are asking the DM to complete the construction of Alternative I' by providing a value for  $\Delta\mathbf{R}_{\mathbf{A}2}$  such that he will be indifferent to I and I'.

We ask the DM to consider the values of the remaining four MOEs  $(\Delta R_{A1},\ \Delta C_{I},\ \Delta C_{OMN})$ , and  $\Delta R_{C})$ , which are the same for I and I'. Given those values, we next ask the DM to consider an increase in the UNREP scenario response time of 0.8 hours, so that the net decrease in response time changes from -3.2 hours to -2.4 hours. The question is what amount of decrease in response time in the repair-at-sea scenario will compensate for the increase in  $\Delta R_{D}$ ? There is a limit to the amount that  $\Delta R_{A2}$  can be decreased. This limit corresponds to the point where  $R_{A2}$  reaches zero and for this case corresponds to the value of  $R_{A2}$  shown in Table V-5, the Base Program response time. Thus, we are asking the value of  $\Delta R_{A2}$  between -13.2 and -118.5 minutes that will compensate for a change in  $\Delta R_{D}$  from -3.2 to -2.4 hours. Let us assume that the response is -21 minutes. This updates Table V-13 to the values shown in Table V-16.

After each response evoked from the DM, his rationale for his response should be recorded. In this particular case, he may argue that

Table V-14 50% INCREMENT PROGRAM TABLEAU

		ARA1 (working minutes)	$^{ m I}$		αv
-24 -23.2 -23.2		(Caapuram	(millions of dollars)	ΔC <sub>OMN</sub> (millions of dollars)	ONC (working minutes)
		0	1.2	-43.5	-19.2
	-23.2	0	1.2	-43.5	-19.2
	-23.2 0		1.2	-43.5	-19.2
(Hypornerical) 11 -23.2	-23.2 0	09-		-43.5	-19.2
(Hypothetical) II''' -23.2 0	-23.2 0	09-	1.1		-19.2
(Real) IV -23.2 0	-23.2 0	09-	1.1	-118.8	-19.2

Table V-15

75% INCREMENT PROGRAM TABLEAU

	II < VI			VI < II		
Alternatives	$\Delta C_{ m I}$ (millions of dollars)	ΔC <sub>OMN</sub> (millions of dollars)	RD (working hours)	R <sub>C</sub> (working minutes)	RA1 (working minutes)	RA2 (working minutes)
(Real) IV	-10.4	-74.7	-23.2	-15.6	-30	9.9
(Hypothetical) IV'	1.8		-23.2	-15.6	-30	9.9
(Hypothetical) IV''	1.8	-118.1		-15.6	-30	9.9
(Hypothetical) IV'''	1.8	-118,1	-24		-30	9.9
(Hypothetical) IV'''	1.8	-118,1	-24	-19.2		9.9
(Hypothetical) IV''''	1.8	-118.1	-24	-19.2	09-	
(Real) II	1.8	-118.1	-24	-19.2	09-	-13.2

Table V-16

25% INCREMENT PROGRAM TABLEAU--UPDATE 1

	1 > 11	11		II > I		$I \sim I$
Alternatives	AR <sub>D</sub> (working hours)	$\Delta_{\mathrm{A2}}^{\mathrm{AA2}}$ (working minutes)	$\begin{array}{c} \Delta R_{AI} \\ \text{(working} \\ \text{minutes)} \end{array}$	$\begin{array}{c} \Delta C_{\rm I} \\ \text{(millions} \\ \text{of dollars)} \end{array}$	ΔC <sub>OMN</sub> (millions of dollars)	$\Delta R_{C}$ (working minutes)
(Real) I	-3.2	-13.2	0	8.0	-36.6	-7.8
(Hypothetical) I'	-2.4	-21	0	8*0	-36.6	8.7-
(Hypothetical) I''	-2.4	0		8.0	-36.6	-7.8
(Hypothetical) I'''	-2.4	0	09-		-36.6	8.7-
(Hypothetical) I''''	-2.4	0	09-	1.0		8.7-
(Real) II	-2.4	0	09-	1.0	-111.9	8.7-

an UNREP response time decrease significantly greater than 2 to 3 hours is needed. At these lower levels, an improvement in response time of about one hour will not be very significant in improving Naval readiness. On the other hand, a change in response time of one hour would be considerably more significant in the repair-at-sea scenario. Thus, for each hour of response time given up in Scenario D, only 10 minutes of improvement in response time in Scenario A is required to compensate. Thus, for a 0.8-hour increase in  $\Delta R_D$  only an 8-minute decrease in  $\Delta R_{A2}$  is required, and  $\Delta R_{A2}$  should go from -13.2 to about -21 minutes.

Since the minimum number of tradeoff assessment have been accomplished, Step 5 consists of comparing Alternative II to I'. We see that II dominates or equals I' in all MOEs except  $R_{A2}$ . Thus, additional tradeoff assessments will be required to establish preference. Note that if the last DMs response had been 0 minutes or more, we would be able to conclude that II was preferred to I' and thus also preferred to I. We would then proceed to Step 1 or Step 6 at the option of the DM. In this case, however, we return to Step 4.

The next tradeoff assessment required is between  $R_{A2}$  and  $R_{A1}$ , the small parts requisition scenario. Here we ask how much should  $\Delta R_{A1}$  be decreased to compensate for a  $\Delta R_{A2}$  increase from -21 to 0? If the DM considers one minute lost in Scenario A2 to be equivalent to 3 minutes of gain in Scenario A1, his response will be to change  $\Delta R_{A1}$  from 0 to -63 minutes. This result updates the tableau from that in Table V-16 to that in Table V-17.

Continuing through this process we finally arrive at the final updated tableau shown in Table V-18. For the final two tradeoff assessments we have assumed that the tradeoff ratio between  $C_{\rm I}$  and  $R_{\rm A1}$  was -0.17 \$M/minute, while between  $C_{\rm OMN}$  and  $C_{\rm I}$  it was -1.5 \$M/\$M. At this point, we note that Alternative II dominates I''' in  $C_{\rm ONN}$  and is equal in all other MOEs. Thus, we conclude that II is preferred to I. Reviewing the MOE values for I and II in Table V-18, we see that the primary tradeoff was between an additional  $C_{\rm OMN}$  savings of \$75 M and a reduced time  $R_{\rm A1}$  of 60 minutes for II versus an additional reduced time  $R_{\rm D}$  of 0.8 hours and a reduced response time  $R_{\rm A2}$  of about 13 minutes

Table V-17

25% INCREMENT PROGRAM TABLEAU--UPDATE 2

	1 > 11	. 11		11 > 1		$I \sim II$
	AR <sub>D</sub> (working hours)	$\begin{array}{c} \Delta R_{A2} \\ \text{(working} \\ \text{minutes)} \end{array}$	$\begin{array}{c} \Delta R_{A1} \\ \text{(working} \\ \text{minutes)} \end{array}$	$\Delta C_{ m I}$ (millions of dollars)	ΔC <sub>OMN</sub> (millions of dollars)	$^{\Delta R}_{\mathbf{C}}$ (working minutes)
(Real) I	-3.2	-13.2	0	0.8	-36.6	-7.8
(Hypothetical) I'	-2.4	-21	0	0.8	9.98-	-7.8
(Hypothetical) I''	-2.4	0	-63	0.8	9*98-	-7.8
(Hypothetical) I'''	-2.4	0	09-		-36.6	-7.8
(Hypothetical) I''''	-2.4	0	09-	0.7		-7.8
(Real) II	-2.4	0	09-	0.7	-111.9	-7.8

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Table V-18
25% INCREMENT PROGRAM TABLEAU--FINAL UPDATE

	11 < 1	11		1 < 11		11 ~ 1	
Alternatives	ΔR <sub>D</sub> (working hours)	ΔR <sub>A2</sub> (working minutes)	$\Delta F_{A1}$ (working minutes)	$\begin{array}{c} \Delta C_{\rm I} \\ (\text{millions} \\ \text{of dollars}) \end{array}$	AC <sub>OMN</sub> (millions of dollars)	AR <sub>C</sub> (working minutes)	DM's Tradeoff Ratios
(Real) I	-3.2	-13.2	0	8*0	-36.6	-7.8	
(Hypothetical) I'	-2.4	-21	0	8.0	-36.6	-7.8	$\frac{\delta R_{A2}}{\delta R_{D}} = -9.7 \text{ min/hr}$
(Hypothetical) I''	-2.4	0	-63	8*0	-36.6	-7.8	$\frac{\delta R_{A1}}{\delta R_{a2}} = -3 \min/\min$
(Hypothetical) I'''	-2.4	0	-60	6.0	-36.6	-7.8	$\frac{\delta C_{\rm I}}{\delta R_{\rm A1}} = -0.17 \ {\rm \$M/min}$
(Hypothetical) I''''	-2.4	0	-60	7.0	-37.2	-7.8	$\frac{\delta C_{OMN}}{\delta C_{I}} = -1.5 \text{ $\%/\$M}$
(Real) II	-2.4	0	-60	0.7	-111.9	-7.8	

for I. The large  $C_{\mbox{OMN}}$  savings predominated for the tradeoff ratios selected for this example.

Step 7, an optional step, consists of expanding the tableau to include a tradeoff assessment between MOE  $C_{OMN}$  and  $R_{C}$ . This provides one more hypothetical alternative and completes the set of tradeoff ratios involving all of the MOEs. From these tradeoff ratios the  $\gamma_{ni}$ 's in Eq. (III-13) can be determined. These represent the implied tradeoff ratios between the last MOE in Table V-18 and each of the other MOEs. The  $\gamma_{ni}$ 's (where n = 6) can be computed recursively from

$$Y_{6i} = -\frac{\delta x_{i+1}}{\delta x_{i}} Y_{6(i+1)}$$
 for  $i = 5, 4, 3, 2, 1$  (V-1)

where  $_{66}^{Y}$  = 1, and  $_{i}$  corresponds to the MOE in the i-th column of Table V-18.

Since negative changes in response time and costs are preferred to positive changes, to rank all alternatives we must minimize Eq. (III-13).

To pursue this example, let us assume that for every \$1M savings in  $^{\rm C}_{\rm OMN}$ , the DM is willing to allow  $^{\rm R}_{\rm C}$  to increase by 5 minutes. This means that his tradeoff ratio is given by

$$\frac{\delta R_{C}}{\delta C_{OMN}} = -5 \min/\$M \qquad . \tag{V-2}$$

The values of  $Y_{ni}$ ,  $\Delta x_i$ ,  $Y_{ni}\Delta x_i$ , and  $g(\Delta x_i)$  for each i and each alternative are shown in Table V-19. From this table we see that indeed g for II is less than g for I. However, the minimum value of g is given for Alternative IV. Thus, IV may be the most preferred alternative. Note the poor relative performance of III. Reviewing each alternative we see that the primary factor in the performance of IV in in  $\Delta R_D$ . It performs more poorly than III in costs, but not by very much. However, it performs better than III in all other MOEs. Alternatives I and II perform significantly better than III in combined costs, and in all other MOEs. Thus, III scores lowest among all alternatives. The tentative relative ranking of all four alternatives is at IV > II > I > III.

Table V-19

OPTIMIZATION WORKSHEET

								1
Alternatives	Parameters	$i = 1$ $\Delta R_D$	$i = 2$ $\Delta R_{A2}$	$i = 3$ $\Delta R_{A1}$	$i = 4$ $\Delta C_{I}$	$i = 5$ $\Delta C_{OMN}$	$i = 6$ $\Delta R_C$	00
	<sup>Y</sup> 6i	37.1 min/hr	3.83 min/min	1.28 min/min	7.5 min/\$M	5 min/\$M	l min/min	
ı	Δx <sub>i</sub> Y <sub>61</sub> Δx <sub>i</sub>	-3.2 -118.7	-13.2 -50.6	0	9.8	-36.6	-7.8 -7.8	-354
11	Δx <sub>1</sub> Y <sub>61</sub> Δx <sub>1</sub>	-2.4	0	8°9 <i>L</i> -	5.3	-111.9	-7.8 -7.8	-728
111	Δx <sub>i</sub> Υ <sub>61</sub> Δx <sub>i</sub>	0	6.6	24 30.7	-11.4	6.9	0	26
IV	Δx <sub>1</sub> Y <sub>61</sub> Δx <sub>1</sub>	-23.2	0	0	0.4	-6.9	-15.6	806-

Note: Relative Ranking--IV > II > I > III.

In Step 8 we must ascertain whether the DM feels that his tradeoff ratios would remain constant over the range of MOE values indicated for the four alternatives. If the DM response is yes, then we infer that the most preferred alternative is IV, the one that minimized the linear function given by Eq. (III-13), and the relative ranking of all alternatives is given above.

If the DM responds that several of his tradeoff ratios may change, depending on the magnitudes of the MOEs, we would proceed to Step 2 with Alternatives II and IV since II is preferred to I, and IV may be preferred to II according the DM's previous tradeoffs. The true relative ranking between II and IV will be determined after the next iteration through the procedure steps. The new tableau for II and IV is shown in Table V-20.

The numerical example is not carried out further since the primary objective of illustrating the procedures of the method has been accomplished. Succeeding iterations ensure that eventually all alternatives are ranked. At the completion of this process, the responses and rationale for each step can be reviewed. Consistency can be checked by reordering MOEs and performing new tradeoff assessments.

A new calculation of the  $\gamma_{ni}$ 's should give results consistent with other previous calculations. Some inconsistencies are bound to be observed. If these are significant enough to affect the final ranking of the alternatives, the inconsistencies should be resolved. This resolution will generally be accomplished by modifying one or several tradeoff assessment responses.

Finally, we can observe that in formulating the initial set of alternative programs, not all possible combinations or levels of project funding are included. After the application of the tradeoff Assessment procedures, the staff acquires an appreciable amount of information on the DM's judgmental tradeoffs. This can be used as a basis for reviewing other alternative possibilities and perhaps adjusting them to obtain an improved alternative. In addition, the information can also form the basis for constructing new alternatives when budget changes occur, or new planning cycles begin.

Table V-20

25% INCREMENTAL, PROGRAM TABLEAU--SECOND ITERATION

$VI \sim II$ $VI \sim IV < VI$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	-111.9 0.7 -7.8 -3.2 0	0.7 -7.8 -3.2 0	-6.9 -3.2 0	-6.9 0.4 -3.2 0	0.4 -15.6 0	-6.9 0.4 -15.6 -23.2 0
11 > 1V	$\Delta R_{A1}$ $\Delta C_{A1}$ (working minutes) of do	-11	0	0	0	0	0
	Alternatives	(Real) II	(Hypothetical) II'	(Hypothetical) II''	(Hypothetical) II'''	(Hypothetical) II'''	(Real) IV

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# Appendix A

A MATHEMATICAL MODEL FOR THE RESOURCE ALLOCATION METHOD

## Appendix A

## A MATHEMATICAL MODEL FOR THE RESOURCE ALLOCATION METHOD

The resource allocation (RA) methodology is based on the assumption that there exists a "preference" or "utility" function f that assigns a real value to each multi-attribute outcome. A multi-attribute outcome is a set of distinct and different outcomes that each have an influence on the preferences of the decision maker (DM). Each individual outcome is an attribute or measure of effectiveness (MOE) of an exploratory development program. Mathematically, an outcome x is defined as a vector whose components consist of the set of MOEs. Thus,

$$x = (x_1, x_2, x_3, ..., x_n)$$
 (A-1)

where x, is the i-th attribute variable.

The preference function f can then be written as

$$f(x) = f(x_1, x_2, x_3, ..., x_n)$$
 (A-2)

This function is such that if one outcome vector x is preferred to another outcome vector y, then f(x) is greater than f(y), and if neither is preferred to the other, then f(x) equals f(y).

Starting with the general form of Eq. (A-2), we will develop a mathematical model of the tradeoff assessment process employed in the RA methodology described in Section III. Let a be a specific value of the vector x. We can form a Taylor series expansion of the function f about the point a. If we ignore quadratic and higher-order terms we obtain

$$f(x) \cong f(a) + \sum_{i=1}^{n} \frac{\partial f(a)}{\partial x_{i}} (x_{i} - a_{i})$$
 (A-3)

where  $\frac{\partial f(a)}{\partial x_i}$  is the partial derivative of f with respect to  $x_i$  evaluated at a.

If f(x) is linear, then Eq. (A-3) is an exact expression for f(x). In any case, it represents a local approximation of f(x) in the vicinity of a. Equation (A-3) can also be written as

$$\hat{f}(x) = f(a) + \frac{\partial f(a)}{\partial x_n} \sum_{i=1}^{n} \frac{\frac{\partial f(a)}{\partial x_i}}{\frac{\partial f(a)}{\partial x_n}} (x_i - a_i)$$
 (A-4)

where  $\hat{f}(x)$  is our approximation to f(x). Let

$$\lambda_{i}(a) = \frac{\frac{\partial f(a)}{\partial x_{i}}}{\frac{\partial f(a)}{\partial x_{n}}} = -\frac{\partial x_{n}}{\partial x_{i}} \bigg|_{f(x)=f(a)}.$$
 (A-5)

The expressions in Eq. (A-5) indicate that  $\lambda_i$  is the incremental tradeoff ratio between attributes  $\mathbf{x}_n$  and  $\mathbf{x}_i$  at point a. It is the incremental amount that  $\mathbf{x}_n$  must be decreased to offset an increase in  $\mathbf{x}_i$ , maintaining a constant value of the preference function. [Note that  $\lambda_n(a)$  is simply equal to 1.] The last expression in Eq. (A-5) indicates that  $\lambda_i$  is the negative of the slope of the constant-value preference curve plotted in the  $(\mathbf{x}_1, \mathbf{x}_n)$  plane. Thus, Eq. (A-4) can be written as

$$\hat{f}(x) = f(a) + \frac{\partial f(a)}{\partial x_n} \sum_{i=1}^{n} \lambda_i(a) (x_i - a_i) . \qquad (A-6)$$

The slope interpretation is illustrated in Figure A-1 for a two-dimensional case (n = 2). The three curves in Figure A-1 are contours of iso-preference or indifference curves. The attributes are such that increasing values are preferred, and thus the preference function value increases in the upper right-hand direction. The point a lies on the curve for which f(x) = f(a). The tangent line to the curve at a have the same slope as the curve, and it is given by  $-\lambda_1$  (a).

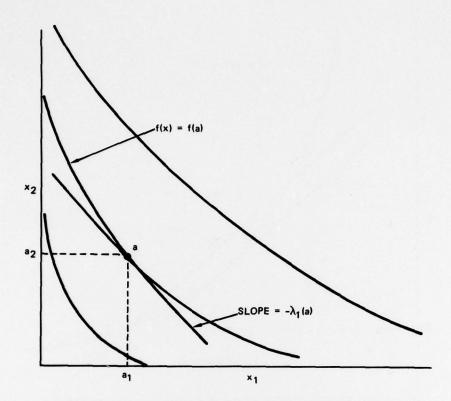


FIGURE A-1 INCREMENTAL TRADEOFF RATIO SLOPE

If we now consider two points a and b such that f(b) = f(a), then the ratio of  $(a_2 - b_2)$  to  $(b_1 - a_1)$ --call it  $Y_1$ --gives the value of  $\lambda_1$  at some intermediate point c on the same indifference curve. This is illustrated in Figure A-2. The value of  $Y_1$  is exactly the ratio we obfrom the DM's tradeoff between two outcome vectors a and b that differ in only two attributes. For well-behaved indifference curves, such as those shown,  $Y_1$  will be a good approximation of  $\lambda_1$  at a or b.

From these considerations, we see that Eq. (A-6) can be written for the two-dimensional case as

$$\hat{f}(x) = f(c) + \frac{\partial f(c)}{\partial x_n} [Y_1(a, b)(x_1 - c_1) + (x_2 - c_2)]$$
 (A-7)

where  $Y_1$ (a, b) indicates that tradeoffs between a and b establish the ratio  $Y_1$ . If Eq. (A-7) were an exact expression for the preference function, we now have enough information from the DM to select a most preferred

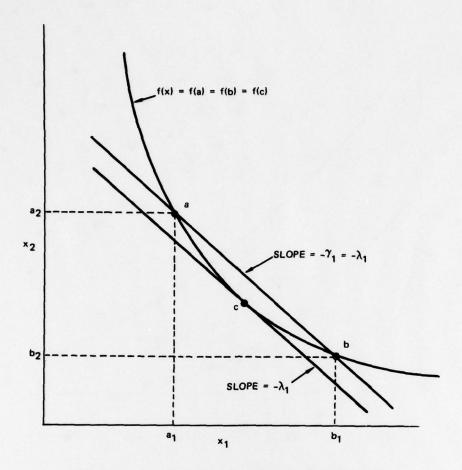


FIGURE A-2 DECISION MAKER'S TRADEOFF SLOPE

outcome from any set of alternate outcomes. To maximize f(x) we need only select the x, from the feasible set of x's, that maximizes

$$g(x) = x_1 Y_1(a, b) + x_2$$
 (A-8)

(The terms in the right-hand side of Eq. (A-8) are the only terms in the right-hand side of Eq. (A-7) that depend on x.) Equation (A-8) indicates that the point c, f(c), and  $\frac{\partial f(c)}{\partial x_n}$  need not be known to solve the maximization problem.

Maximizing Eq. (A-8) is shown graphically in Figure A-3. We simply slide the tradeoff line up to the right until we have swept by all but one feasible outcome. This outcome, e, maximizes Eqs. (A-8) and (A-7). In case Eq. (A-7) is not exact, the feasible outcome maximizing Eq. (A-8)

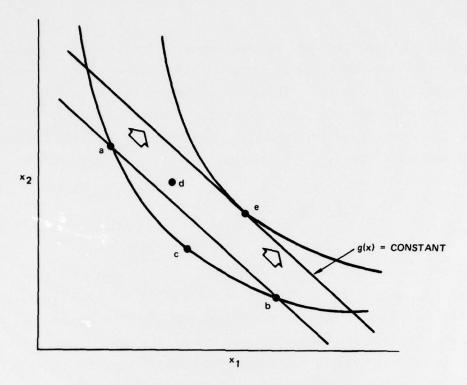


FIGURE A-3 MAXIMIZING THE LINEAR FUNCTION

is still likely to be most preferred and should be selected as one of the pair of outcomes for the next iteration of preference assessments.

The above analysis can be generalized to higher-dimensional problems as follows. First, we generalize the incremental tradeoff ratio to include tradeoffs between any pair of attributes  $x_i$  and  $x_j$ , where  $1 \le n$  and  $j \le n$ . Thus, let

$$\lambda_{ji}(a) = -\frac{\partial x_j}{\partial x_i} \bigg|_{f(x)=f(a)}$$
 (A-9)

where  $\lambda_{ii}(a)$  = 1, and  $\lambda_{ni}(a)$  is equal to  $\lambda_{i}(a)$  defined by Eq. (A-5). It can be shown that

$$\lambda_{ni}(a) = \prod_{k=i}^{n-1} \lambda_{(k+1)k}(a)$$
 (A-10)

Equation (A-10) states that we can obtain the tradeoff ratios between any two attributes as the product of tradeoff ratios between adjacent attributes.

Similarly, the tradeoff ratios between attributes for outcome pairs, the Y's, can be generalized by first defining the concept of attribute linking pairs of outcomes. Thus, let  $a^i$  and  $a^j$  be two outcome vectors all of whose attribute values, except the i-th and j-th attributes, are equal, and whose i and j attribute values are such that the DM is indifferent between  $a^i$  and  $a^j$ . We then define

$$Y_{ji}(a^{j}, a^{i}) = \frac{a_{i}^{i} - a_{j}^{j}}{a_{i}^{j} - a_{i}^{i}}$$
 (A-11)

The notation in Eq. (A-11) is cumbersome, but the idea is simple. The numerator is simply the difference between the j-th attribute values for the attribute linking pair of vectors a and a . The denominator is the difference between the i-th attribute values for the same vector pair (taken in reverse order). When the DM is indifferent between the attribute linking pairs of outcome vectors, Eq. (A-11) is exactly the tradeoff information obtained from the DM tradeoff assessments, and the attribute-linking outcome pairs are the pairs of hypothetical outcomes constructed in the process.

The expression for  $\gamma_{ni}$  in terms of  $\gamma_{(k+1)k}$  for various values of k is exactly the same form as Eq. (A-10), but  $\gamma_{ni}$  is a function of a sequence of outcome vectors made up of attribute linking pairs:

$$Y_{ni}(a^n, ..., a^{i+1}, a^i) = \prod_{k=i}^{n-1} Y_{(k+1)k}(a^{k+1}, a^k)$$
 (A-12)

The equation for  $\hat{f}(x)$  becomes

$$\hat{f}(x) = f(c) + \frac{\partial f(c)}{\partial x_n} \sum_{i=1}^{n} \gamma_{ni}(x_i - c_i)$$
 (A-13)

where the functional dependence of  $Y_{ni}$  on the set of attribute-linking vector pairs has been suppressed to ease the notation problem.

Equation (A-13) can be interpreted in the following way. The dot product of the Y-vector and the outcome vector, (x - c), set equal to a

constant is the equation of a hyperplane in the n-dimensional space of attributes. [This dot produce is given by the summation term on the right-hand side of Eq. (A-13).] Similarly, the  $\lambda$ -vector dot product with the outcome vector, x - c, is also a hyperplane. The Y-hyperplane is a tangent plane to the indifference hypersurface in n-dimensional space at some point c. The point c is that point for which the Y-hyperplane containing the n outcome vectors at which the Y's were determined, is parallel to the  $\lambda$ -hyperplane. Note that the n outcome vectors lie on the same indifference hypersurface as c. Thus, Eq. (A-13) is a linear model for f(x) in the neighborhood of c.

As before, to select a feasible outcome vector for the next iteration of tradeoff assessments we maximize the function

$$g(x) = \sum_{i=1}^{n} Y_{ni} x_{i}$$
 (A-14)

Again, this function is independent of c,  $\frac{\partial f(c)}{\partial x}$ , and f(c), so that c does not need to be determined explicitly.

Appendix B

AN SR-52 PROGRAM FOR COMPUTING EXPECTED RESPONSE TIME

## Appendix B

### AN SR-52 PROGRAM FOR COMPUTING EXPECTED RESPONSE TIME

To facilitate the calculation of expected response times according to the model presented in Section IV-A, a program was written for the Texas Instruments SR-52 hand calculator. Memory locations 00 through 19, and 98 and 99 are used to store the responsiveness parameters according to the order shown in Table V-2. Note that the sums of several responsiveness parameters are stored in certain locations.

The parameter values for each case can either be input by hand or stored on a second magnetic card as a program to load the memory. This works only for loading memory locations 00 through 19. Locations 98 and 99 must be loaded by hand after running the program that loads 00 through 19.

Table B-1
SR-52 PROGRAM LISTING

1.	2nd LBL	31.	1	61.	9
2.	A	32.	9	62.	8
3.	(	33.	)	63.	)
4.	RCL	34.	2nd RTN	64.	X
5.	1	35.	2nd LBL	65.	(
6.	7	36.	С	66.	RCL
7.	+	37.	(	67.	1
8.	RCL	38.	2	68.	4
9.	1	39.	-	69.	-
10.	8	40.	RCL	70.	RCL
11.	-	41.	9	71.	1
12.	RCL	42.	9	72.	6
13.	1	43.	x	73.	)
14.	7	44.	RCL	74.	+
15.	x	45.	9	75.	(
16.	RCL	46.	8	76.	С
17.	1	47.	)	77.	-
18.	8	48.	2nd RTN	78.	A
19.	)	49.	2nd LBL	79.	X
20.	2nd RTN	50	E '	80.	RCL
21.	2nd LBL	51.	(	81.	9
22.	В	52.	1	82.	9
23.	(	53.	-	83.	X
24.	2	54.	A	84.	(
25.	-	55.	)	85.	RCL
26.	RCL	56.	X	86.	1
27.	9	57.	(	87.	9
28.	9	58.	1	88.	-
29.	x	59.	•	89.	RCL
30.	RCL	60.	RCL	90.	9

Table B-1 (Continued)

91. 8
93. ) 94. X 95. ( 96. RCL 97. 0 98. 0 128. RCL 129. 1 158. A 129. 1 100. RCL 101. 0 102. 1 103. ) 104. + 105. B 106. X 107. ( 108. RCL 109. 0 1109. 0 1100. RCL 1101. 0 1101. 0 1102. 1 1103. ) 1104. + 1105. B 1106. X 1107. ( 1107. ( 1107. ( 1108. RCL 1109. 0 1109
94. X       124. RCL       154. X         95. (       125. 1       155. (         96. RCL       126. 1       156. 1         97. 0       127. +       157         98. 0       128. RCL       158. A         99. +       129. 1       159. )         100. RCL       130. 5       160. X         101. 0       131. )       161. (         102. 1       132. +       162. RCL         103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
95. ( 125. 1 155. ( 96. RCL 126. 1 156. 1 156. 1 157 98. 0 128. RCL 158. A 99. + 129. 1 159. ) 100. RCL 130. 5 160. X 101. 0 131. ) 161. ( 102. 1 132. + 162. RCL 103. ) 133. ( 163. 0 104. + 134. 1 164. 2 105. B 135 165. + 106. X 137. 1 167. 0 108. RCL 138. 9 168. 4 109. 0 139. ) 169. + 110. 3 140. X 170. RCL
96. RCL       126. 1       156. 1         97. 0       127. +       157         98. 0       128. RCL       158. A         99. +       129. 1       159. )         100. RCL       130. 5       160. X         101. 0       131. )       161. (         102. 1       132. +       162. RCL         103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
97. 0 127. + 157 98. 0 128. RCL 158. A 99. + 129. 1 159. ) 100. RCL 130. 5 160. X 101. 0 131. ) 161. ( 102. 1 132. + 162. RCL 103. ) 133. ( 163. 0 104. + 134. 1 164. 2 105. B 135 165. + 106. X 136. RCL 166. RCL 107. ( 137. 1 167. 0 108. RCL 138. 9 169. + 110. 3 140. X 170. RCL
98. 0 128. RCL 158. A 99. + 129. 1 159. ) 100. RCL 130. 5 160. X 101. 0 131. ) 161. ( 102. 1 132. + 162. RCL 103. ) 133. ( 163. 0 104. + 134. 1 164. 2 105. B 135 165. + 106. X 136. RCL 166. RCL 107. ( 137. 1 167. 0 108. RCL 138. 9 168. 4 109. 0 139. ) 169. + 110. 3 140. X 170. RCL
99. + 129. 1 159. ) 100. RCL 130. 5 160. X 101. 0 131. ) 161. ( 102. 1 132. + 162. RCL 103. ) 133. ( 163. 0 104. + 134. 1 164. 2 105. B 135 165. + 106. X 136. RCL 166. RCL 107. ( 137. 1 167. 0 108. RCL 138. 9 168. 4 109. 0 139. ) 169. + 110. 3 140. X 170. RCL
100. RCL       130. 5       160. X         101. 0       131. )       161. (         102. 1       132. +       162. RCL         103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
101. 0       131. )       161. (         102. 1       132. +       162. RCI         103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCI         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCI
102. 1       132. +       162. RCL         103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
103. )       133. (       163. 0         104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
104. +       134. 1       164. 2         105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
105. B       135       165. +         106. X       136. RCL       166. RCL         107. (       137. 1       167. 0         108. RCL       138. 9       168. 4         109. 0       139. )       169. +         110. 3       140. X       170. RCL
106. X     136. RCL     166. RCL       107. (     137. 1     167. 0       108. RCL     138. 9     168. 4       109. 0     139. )     169. +       110. 3     140. X     170. RCL
107. (     137. 1     167. 0       108. RCL     138. 9     168. 4       109. 0     139. )     169. +       110. 3     140. X     170. RCI
108. RCL 138. 9 168. 4 109. 0 139. ) 169. + 110. 3 140. X 170. RCL
109. 0 139. ) 169. + 110. 3 140. X 170. RCI
110. 3 140. X 170. RCI
111. + 141. A 171. 0
112. RCL 142. X 172. 6
113. 0 143. ( 173. +
114. 5 144. RCL 174. RCI
115. + 145. 1 175. 0
116. RCL 146. 3 176. 8
117. 0 147 - 177. +
118. 7 148. RCL 178. RCI
119. + 149. 1 179. 1
120. RCL 150. 5 180. 0

Table B-1 (Concluded)

181.	+	188.	6	195.	7
182.	RCL	189.	)	196.	X
183.	1	190.	•	197.	RCL
184.	2	191.	В	198.	0
185.	+	192.	x	199.	1
186.	RCL	193.	RCL	200.	-
187.	1	194.	1	201.	HLT
		Annual Control of the			